Research on Reducing the Resonance Frequency of Low Frequency Metamaterials

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***Abstract*—Metamaterials, as a new type of material for manufacturing electromagnetic and electronic devices, have become the focus of many studies. It can be used in many disciplines, such as chemistry, physics, materials science, optoelectronics, semiconductor science, and equipment manufacturing, and are currently a cutting-edge and strategically significant research topic. Electromagnetic metamaterials are a new type of artificial composite electromagnetic material with controllable dielectric constant and magnetic permeability. This paper establishes a theoretical framework for metamaterials, divides electromagnetic wave media into four quadrants using constitutive parameters, and introduces the calculation method of effective magnetic permeability. This paper is based on a square planar spiral structure. In order to reduce the resonant frequency, a quantitative analysis was conducted on the influence of double-layer mirror structure, patch resistance, and different stacking structures on the resonant frequency. The reference resonant frequency was reduced from 9.6MHz to 7.6MHz, providing a solid foundation for subsequent research.**

Keywords: Electromagnetic metamaterial, Resonant frequency, Planar spiral inductor, Negative refraction characteristics, s-parameter.

I. INTRODUCTION

Metamaterials were first proposed by former Soviet theoretical physicist Viserago in 1968[1][2]. Veselago speculated that the dielectric constant and magnetic permeability can be negative, but it is only limited to theoretical derivation. Until 1996, Pendry of the Royal Institute of Technology in the UK proposed the structure of metal thin wires[3], which achieve negative dielectric constant by periodically arranging them in the microwave band. Subsequently, in 1999, an open resonant ring structure was proposed[4], which achieved negative magnetic permeability through periodic arrangement of metal open resonant rings. In 2001, American physicist R.A. Shelby and others verified the existence of metamaterials through prism experiments, and observed the phenomenon of ”negative refraction”[5] for the first time.

According to the working frequency band and application field of electromagnetic metamaterials, they can be divided into the following three categories: Low frequency metamaterials, microwave and optical metamaterials, radio frequency metamaterials. This paper mainly involves low-frequency metamaterials,it working frequency band is lower than MHZ, mainly used in magnetic shielding[6][7], magnetic resonance imaging[8][9][10], and magnetic resonance radio energy transmission systems[11]. Compared to ordinary metal layer magnetic shielding, metamaterials can be frequency selective shielding. In 2021, Professor Pkharel Ramesh from Kyushu University in Japan proposed a stacked split ring resonator metamaterial for radio energy transmission to increase transmission power. Reference [12] proposes a hybrid electromagnetic metamaterial plate, which combines two types of electromagnetic metamaterial structures with negative and zero permeability, respectively, and is applied to the transmission between the receiving coil and the transmitting coil, increasing the transmission efficiency from 8.7% to 47%.Besnoff et al. proposed a near zero permeability electromagnetic metamaterial[13], which is placed outside the resonant coil in a WPT system, effectively reducing the magnetic field intensity outside the system. Ma Yanbing from the University of Electronic Science and Technology proposed a dual frequency negative permeability electromagnetic metamaterial[11] based on the Koch fractal structure, which resonates at frequencies of 4.57GHz and 12.82GHz for radio energy transmission.

This article mainly studies electromagnetic metamaterials in low-frequency near-field environments, and compares and analyzes four commonly used metamaterial research and design methods. A electromagnetic metamaterial with FR-4 as the substrate and spiral coil metal wires as the resonant unit is proposed. The proposed model is used to quantitatively study the effects of design parameters and physical connections on the resonant frequency. The most significant feature of low-frequency near-field electromagnetic systems is their low operating frequency and the overall spatial size of the system is much smaller than the wavelength. The electromagnetic field meets the quasi-static field conditions, and the magnetic and electric fields are almost coupled throughout the space. Electromagnetic energy hardly propagates outward in the form of waves. Therefore, ”single negative” metamaterials with negative dielectric constant and permeability can be used to

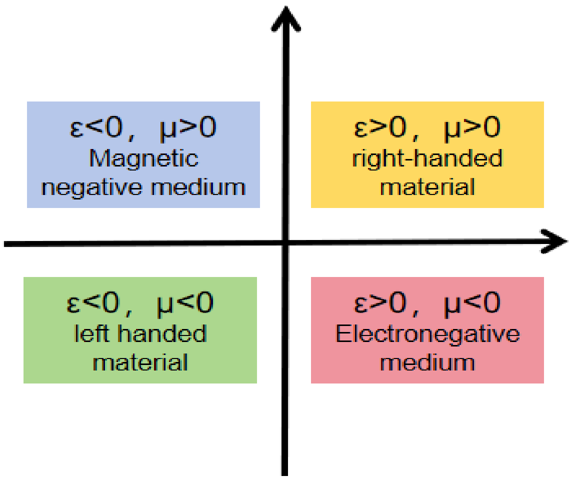

Fig. 1: Metamaterial classification diagram

regulate the near-field electromagnetic field.

II. Basic Theory of Electromagnetic Metamaterials

When electromagnetic waves propagate in it, the direction of the electric field, magnetic field, and electromagnetic wave vector satisfy the left-handed rule, which is completely different from the right-handed rule that is commonly used in electromagnetic media. As a result, the direction of electromagnetic wave energy flow is opposite and parallel to the direction of electromagnetic wave propagation, and the group velocity direction of electromagnetic waves is opposite to the phase velocity direction, thus possessing the characteristics of reverse waves and generating a reverse Doppler effect. In theoretical research, electromagnetic materials are often spatially divided into four parts, as shown in Fig.1, where the abscissa represents the dielectric constant and the ordinate represents the magnetic permeability. In the third quadrant, left-handed materials with negative values for both parameters are represented, exhibiting electromagnetic properties such as negative refractive index, inverse Cherenkov radiation effect, and inverse Doppler effect.

A. *Negative refractive index characteristic*

When an electromagnetic wave is incident on the surface of a left-handed material, the characteristics of the left-handed material can cause a phase change in the electromagnetic wave, resulting in the distribution of refracted and reflected waves on both sides of the normal.

Derive from the complex form of Maxwell's equations:

$$k \times \mathbf{E} = 0 \tag{2.1}$$

$$k \times \mathbf{H} = 0 \tag{2.2}$$

$$k \times \mathbf{E} = \mu\omega\mathbf{H} \tag{2.3}$$

$$k \times \mathbf{H} = -\mu\omega\mathbf{H} \tag{2.4}$$

Where E represents the electric field strength, H represents the magnetic field strength, Use k to perform vector product on the left and right sides of equation (2.3):

$$k \times (\ k \times \mathbf{E}) = \ k(\ k \cdot \mathbf{E}) - (\ k \cdot\ k)\mathbf{E} = \mu\omega\ k \times \mathbf{H} \tag{2.5}$$

Substitute equations (2.1) and (2.2) into (2.5):

$$(\ k \cdot\ k - \mu\epsilon\omega^2)\mathbf{E} = 0 \tag{2.6}$$

Due to the existence of non zero solutions for E, then:

$$k \cdot\ k - \mu\epsilon\omega^2 = 0 \tag{2.7}$$

It can be written as:

$$\frac{\omega^2}{k^2} = \frac{1}{\mu\epsilon} = \frac{1}{\mu_0\epsilon_0\mu_r\epsilon_r} = \frac{c^2}{n^2} \tag{2.8}$$

Among them, c is the speed of light, ϵ_0 is the vacuum dielectric constant, μ_0 is the vacuum magnetic permeability; $V = \omega/k$, v is the propagation speed of electromagnetic waves in the medium; $n = \mu_r\epsilon_r$,n is the refractive index, μ_r is the relative magnetic permeability, ϵ_r is the relative dielectric constant. Solution:

$$n = \pm\sqrt{\mu_r\epsilon_r} \tag{2.9}$$

According to the generalized Snell's law, it can be concluded that:

$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1} = \frac{p_2\left|\sqrt{\mu_2\epsilon_2}\right|}{p_1\left|\sqrt{\mu_1\epsilon_1}\right|} \tag{2.10}$$

Where θ_1 is the incident angle of the electromagnetic wave in the incident medium, θ_2 is the refractive angle of the electromagnetic wave entering the refracting medium from the incident medium. The refractive index image represented by formula (2.10) is shown in Fig.2.

B. *Smith algorithm*

In 2002, Smith et al.[12] proposed to calculate the dielectric constant and equivalent permeability of electromagnetic metamaterials based on the reflection coefficient S11 and transmission coefficient S21 generated by plane waves incident on the surface of electromagnetic metamaterials. In 2005, they improved on the original method and proposed an algorithm for non-uniform materials [13]. Smith et al. proposed a method for extracting constitutive parameters of electromagnetic metamaterials, which is simple in calculation and accurate in results, making it the most commonly used method for studying electromagnetic metamaterials.

According to electromagnetic field theory, the transmission matrix can be obtained from the transmission line equation:

$$T = cos^2(nkd) + sin^2(nkd) \tag{2.11}$$

In equation (2.11), n is the refractive index, k is the wave number, and d is the thickness of the medium through which the electromagnetic wave passes during propagation. Due to the fact that the scattering parameters S and phase parameters can be obtained through numerical simulation software, the

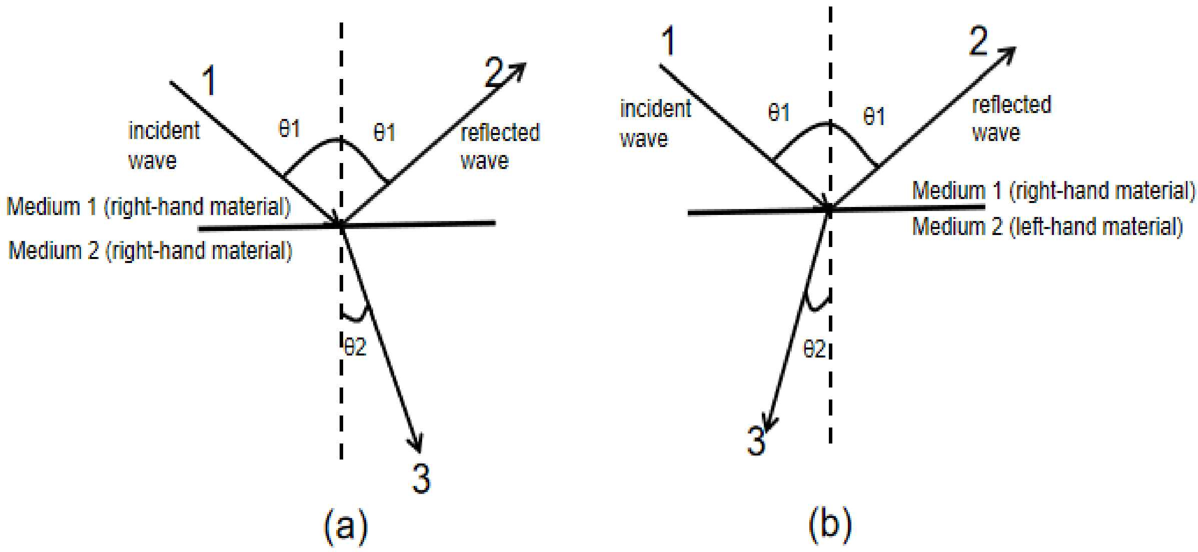

Fig. 2: Schematic diagram of electromagnetic incident on layered surfaces of different media (a) Refraction phenomenon of right-handed materials (b) Refraction phenomenon of left-handed materials

transfer matrix T can be obtained from the scattering parameters s.

$$S_{21} = \frac{2}{T_{11} + T_{21} + (ikT_{12} + \frac{T_{21}}{ik})} \tag{2.12}$$

$$S_{11} = \frac{T_{11} - T_{12} + (ikT_{12} - \frac{T_{21}}{ik})}{T_{11} + T_{22} + (ikT_{12} + \frac{T21}{ik})} \tag{2.13}$$

$$S_{22} = \frac{T_{22} - T_{11} + (ikT_{12} - \frac{T_{21}}{ik})}{T_{11} + T_{22} + (ikT_{12} + \frac{T21}{ik})} \quad (2.14$$

$$S_{12} = \frac{2det(T)}{T_{11} + T_{22} + (ikT_{12} + \frac{T21}{ik})} \tag{2.15}$$

According to the theory of symmetric structure and circuit theory, equations (2.12) to (2.15) can be simplified as follows: Substitute (2.11) into (2.16) and (2.17):

$$S_{11} = S_{22} = \frac{\frac{1}{2}(\frac{T_{21}}{ik} - ikT_{21})}{T_s + \frac{1}{2}(ikT_{12} + \frac{T_{21}}{ik})} \tag{2.16}$$

$$S_{21} = S_{12} + \frac{1}{T_S + \frac{1}{2}(ikT_{12} + \frac{T_{21}}{ik})} \tag{2.17}$$

Transform the above two formulas to obtain the refractive index n and wave impedance z:

$$n = \frac{1}{kd}cos^{-1}[\frac{1}{2S_{21}}(1 - S_{11+}^2 S_{21}^2)] \tag{2.18}$$

$$Z = \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} \tag{2.19}$$

After inverting the refractive index and intrinsic impedance of electromagnetic metamaterials, their constitutive parameters can be calculated according to equation (2.20),(2.21).

$$\mu = nz \tag{2.20}$$

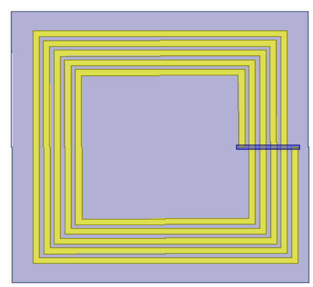

Fig. 3: Resonant Elements of a Square Spiral Structure

$$\epsilon = n/z \tag{2.21}$$

III. Research on Magnetic Negative Metamaterialss

This article mainly studies metamaterials working in the terahertz range. In terms of layout simplicity, square spiral inductors have become the preferred choice for various layouts such as square, hexagonal, octagonal, and circular. Among various spiral structures, square spiral structures have a higher quality factor Q[14]. As shown in Fig.3, the metamaterial resonant unit consists of a square spiral copper metal and a FR4 substrate, with a resonant frequency of 9.2MHz as shown in Fig.4. When the magnetic field is perpendicular to the resonant unit, the copper metal of the magnetic resonance body in this structure produces an inductance effect, and adjacent copper metals form a gap capacitance, which meets the conditions for LC oscillation circuit.

The key to designing high-performance low-frequency metamaterials lies in designing metamaterial elements with lower resonant frequencies and ideal resonant strengths. The

TABLE I: Parameters of metamaterial elements

parameter	numerical value
Base length/mm	140
substrate thickness/mm	1
Outer diameter of coil/mm	120
Inner diameter of coil/mm	74
line width/mm	3
Coil gap/mm	2
Coil thickness/mm	0.035
Coil turns	5
External capacitance/pF	40
Relative dielectric constant of substrate	4.4

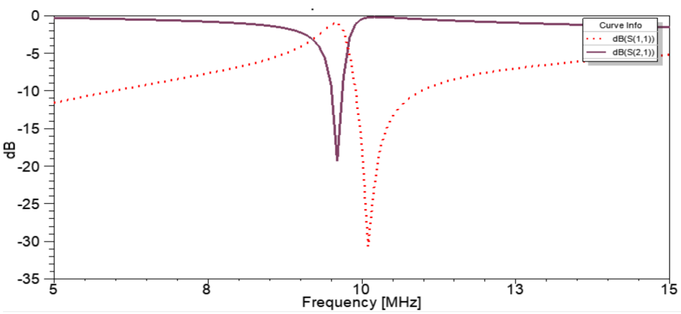

Fig. 4: The S-parameters of the square spiral structure resonant unit

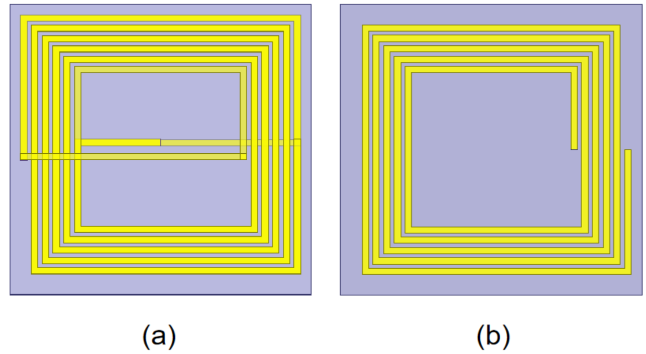

Fig. 5: (a) Resonant elements of a double layered square spiral structure(opposite polarity) (b) Resonant elements of a double layered square spiral structure(Same polarity)

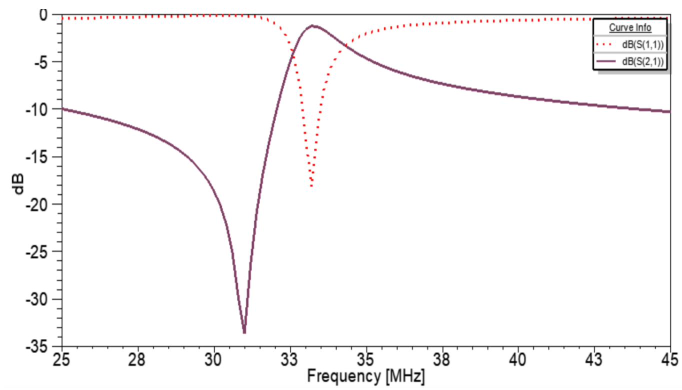

Fig. 6: The S-parameter of a double-layer square spiral structure resonant element with the same polarity

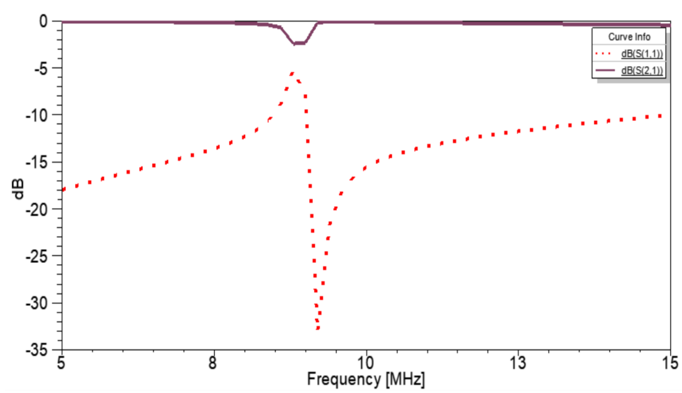

Fig. 7: The S-parameter of a double-layer square spiral structure resonant element with opposite polarity

resonant frequencies and resonant strengths of metamaterial elements are related to the structure and arrangement of the elements themselves.This article will select a square spiral structure resonant unit as shown in Fig.3 to quantitatively analyze the factors that affect the resonant frequency. The various parameters of this structure are shown in Table 1.

A. The Influence of Double Layer Mirror Structure on Resonant Frequency

In order to achieve a lower operating frequency while maintaining a smaller primitive size, this paper proposes a double-layer mirror structure. The double-layer mirror structure is shown in Fig.5(a) and Fig.5(b), which adds the same spiral structure on the other side of the substrate on the structure shown in Fig.3. The two metal spiral structures rotate in opposite directions and are connected through vias at the outermost ring, and are connected in series with the lumped capacitor at the innermost ring.

According to simulation, after using a dual layer mirror structure with opposite polarity as shown in the Fig.7, the resonant frequency is reduced from 9.6MHz to 8.8MHz; After using a dual layer mirror structure with the same polarity as the elements as shown in the Fig.6, the resonant frequency increased from 9.6MHz to 33.2MHz. In order to increase the inductance and reduce the resonant frequency, the rotation directions of the upper and lower layers must be opposite, and the two must be connected in series. This conclusion is consistent with the simulation results in reference [6].

In the dual layer mirror structure with opposite polarity, the stray capacitance of the spiral structure itself is smaller than the total capacitance value, and the total capacitance

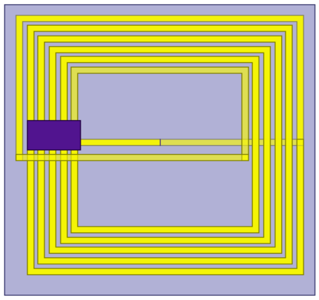

Fig. 8: Double layer resonant element with opposite polarity and added patch capacitor

dominates. The spiral with opposite rotation direction significantly increases the inductance of the primitive. The fundamental inductance and lumped capacitance are the main factors affecting the fundamental resonant frequency, so they are significantly lower than the self resonant frequency of the helix itself. Apply a vertical time-varying magnetic field excitation to the primitive shown in Fig.5(b). Assuming that at a certain moment the magnetic field direction is upward and the magnetic field size increases with time, the upper structure generates a clockwise induced current, and positive charges gather in the outer circle of the helix; The lower spiral also generates a clockwise induced current, but positive charges accumulate in the inner circle.In order to obtain a large inductance and reduce the harmonic frequency, the rotation of the two spirals must be opposite. If the rotation direction of two spirals is the same, then they are in parallel relationship. At this point, the overall inductance of the primitive is much smaller than the inductance when the spirals are connected in series, so the resonance frequency is higher.

B. The Influence of Chip Capacitor on Resonant Frequency

Without changing other parameters of the resonant unit, adjust the capacitance value and size of the external capacitor, as well as the distance between the external capacitor and the resonant unit, to change the resonant frequency and verify the feasibility of the scheme.The simulation results show that as the external capacitance increases, the resonant frequency decreases; As the distance between the external capacitor and the element decreases, the resonant frequency decreases. The distance between the external capacitor and the resonant element is the dominant factor in reducing the resonant frequency. When the distance is greater than 0.1mm, increasing the capacitance value of the external capacitor has almost no effect on the resonant frequency.In a double layer resonant element with the same polarity, the distance is reduced from 0.5mm to 0.2mm, and the resonant frequency is reduced from 33MHz to 30MHz. After the distance is reduced to a certain extent, the resonant frequency undergoes irregular changes.

Based on the above analysis, this article mainly studies the influence of the distance between the patch capacitor and the resonant component, as well as the length of the patch capacitor itself, on the resonant frequency. The parameters of the added chip resistor are shown in Table 2. As shown in Fig.9, as the distance decreases, the resonant frequency point significantly decreases. For the primitive shown in Figure 3, the lowest resonant frequency of 8.9MHz was obtained at a distance of 0.002mm. For the primitive shown in Figure 5 (a), the lowest resonance frequency of 8.2MHz was obtained at a distance of 0.002mm. By reducing the distance by 1mm, the frequency has decreased by approximately 7.3%.As shown in Fig.10, as the length of the patch capacitor increases, the resonant frequency point significantly decreases. The length of the patch capacitor increased from 14mm to 28mm, and for the

TABLE II: Parameters of chip capacitors

parameter	numerical value
length/mm	24
width/mm	14

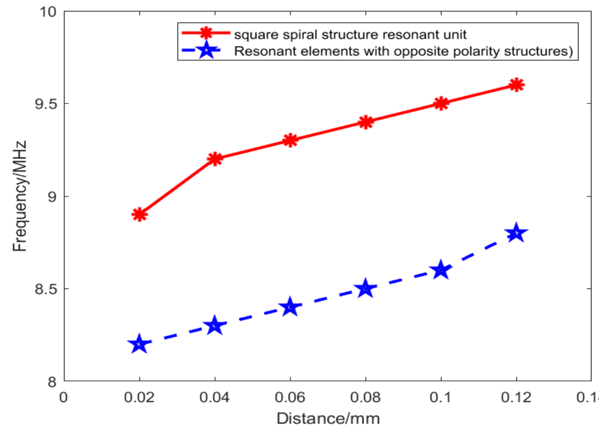

Fig. 9: The Influence of the Distance between the Chip Capacitors and the Resonant Element on the Resonant Frequency

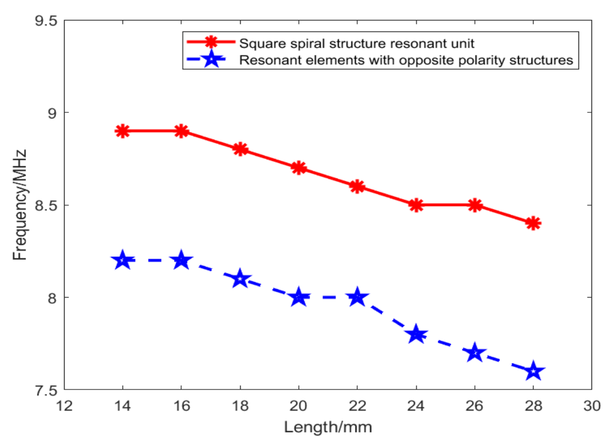

Fig. 10: The Influence of the Length of Chip Capacitors on the Resonant Frequency

element shown in Fig.3, the resonant frequency decreased from 8.9MHz to 8.2MHz. For the primitive shown in Fig.5 (a), the resonant frequency was reduced from 8.2MHz to 7.6MHz, and the resonant frequency was approximately reduced by 7.3%.

C. The Influence of Vertical Stacking Structure on Resonant Frequency

The arrangement of elements and their coupling effects can also affect the properties of metamaterials. The metamaterial studied in this article is at the centimeter level, and all the elements are closely arranged, so there is electromagnetic coupling in the form of mutual inductance between the elements. Assuming that all elements generate similar responses when subjected to an external magnetic field, the instantaneous induced current of each element is the same. There are mainly horizontal coupling between horizontally placed elements and vertical coupling between vertically placed elements.

When the primitive is placed horizontally, the direction of the induced current on the left and right sides of the boundary between the two primitives is opposite. The direction of the induced magnetic field generated by the instantaneous induced current of the primitive itself and the direction of the induced magnetic field generated by the instantaneous induced current of the surrounding primitives are different, and the two cancel out each other. At this time, the mutual inductance between the primitives is negative. When the primitive is placed vertically, the direction of the induced current on the left and right sides of the boundary between the two primitives is the same. The direction of the induced magnetic field generated by the instantaneous induced current of the primitive itself and the direction of the induced magnetic field generated by the instantaneous induced current of the surrounding primitives are the same, and the two strengthen each other. At this time, the mutual inductance between the primitives is positive. According to the expression of resonant frequency, it can be seen that the horizontal arrangement of elements increases the resonant frequency of metamaterials, while the vertical stacking of elements decreases the resonant frequency of metamaterials.

Vertically tightly stacked primitives reduce the working frequency of metamaterials, increase resonance strength, and thus improve the electromagnetic performance of metamaterials. As shown in Fig.11, for the primitive shown in Fig.3, the stacking structure reduces the resonant frequency from 9.6MHz to 7.8MHz. The frequency has decreased by approximately 18.8%.

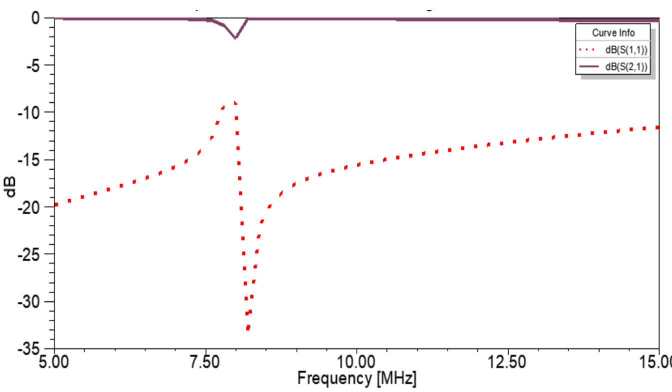

Fig. 11: S-parameters of resonant elements in vertically stacked metamaterials

IV. CONCLUSION

The three methods proposed in this article have all reduced the resonant frequency to a certain extent, achieving the goal of reducing the resonant frequency. A helical structure with opposite polarity can reduce the frequency by 0.8 MHz; The distance between the patch capacitor and the resonant element decreases from 0.1mm to 0.02mm, and the frequency can be reduced by approximately 0.8MHz; Using a stacked structure can reduce the frequency by approximately 1.8 MHz. Subsequent work can focus on how to reduce the frequency below MHZ. At present, low-frequency metamaterials mostly use resonant metamaterials, and the mechanism of electromagnetic resonance determines that these types of metamaterials usually can only work within a narrow frequency band range. In the future, it is necessary to study broadband metamaterials within a broadband range.

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