A Backstepping Sliding Mode Controller Design for Spacecraft Formation Flying

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A BACKSTEPPING SLIDING MODE CONTROLLER DESIGN FOR SPACECRAFT FORMATION FLYING

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In this paper, a backstepping sliding mode controller is developed for tracking control of spacecraft formation flying on elliptical orbits. The controller is designed in accordance with the nonlinear model of relative motion, and combines the advantages of backstepping and sliding mode control techniques. After applying the backstepping method to incorporate the tracking errors and Lyapunov functions, a sliding mode controller is developed to guarantee the Lyapunov stability, handling all nonlinearities, robustness against uncertainties and tracking the desired trajectory. It is supposed that the leader and follower spacecraft are in a low Earth orbit while J2 perturbation and atmospheric drag are considered as external disturbances. The performance of the proposed controller in tracking the desired formation is compared to a sliding mode controller. Simulation results confirm the effectiveness of the proposed controller.

INTRODUCTION

Spacecraft formation flying (SFF) has increasingly attracted attention during the last few decades. Using this method, a large and expensive spacecraft can be replaced with a number of smaller, less expensive and cooperative spacecraft which work as an integrated unit and fulfill the purpose of the mission. Besides the simpler design and faster launching process, the main advantage of this system lies in reliability and flexibility which makes novel and innovative applications in space and the Earth science missions including observation of the Earth and its atmosphere, geodesy, deep space imaging with high resolution, in-orbit servicing and spacecraft maintenance[1]. A common method for implementation of SFF is *Leader/Follower* architecture. Based on this method, one spacecraft is controlled as a leader in a reference orbit while other spacecraft -as followers- adjust their positions relative to the leader and track the desired relative trajectory. In this study, attention is confined to control problem of SFF on elliptical orbits. A robust and precise nonlinear controller is necessary for spacecraft formation process, because it has nonlinear couple dynamics subjected to external disturbances.

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It is well known that sliding mode control (SMC) is a robust method to control nonlinear systems which has satisfactory performance to keep the systems insensitive to the uncertainties and disturbances. An effective robust method for satellite control using a sliding mode controller was presented based on Hill's equations. Parameters including the equivalent damping ratio, bandwidth and thrust were also estimated to minimize the fuel cost[2]. Hui et al. designed a low-level SMC for SFF based on nonlinear relative dynamics of circular reference orbit in another study[3]. They used the controller to control leader, follower and entire-formation maneuvering in low-Earth orbits. Second order sliding mode controller was also applied to control of spacecraft relative translation by Pukdeboon[4]. In another research, Hui and Li presented a terminal SMC to reduce fuel consumption, track desired trajectory and accomplish formation reconfiguration in a proper time[5]. An adaptive terminal sliding mode controller was designed for spacecraft formation flying by Wang and Sun, in which Leader/Follower architecture was considered and the convergence of the desired trajectory to a neighborhood in finite time was proved analytically[6]. In some studies, adaptive SMC was used to control the relative position and attitude of spacecraft in a formation[7,8]. Bae and Kim designed an adaptive controller based on neural network to compensate the modeling error, external disturbance and nonlinearities to improve the performance of the sliding mode controller[7].

Backstepping controller as a nonlinear controller is a recursive process to control nonlinear systems. It is based on Lyapunov theory and can be designed with a systematic strategy. The system is initially represented in small subsystems, then by a recursive approach virtual Lyapunov-based control signals are determined for each subsystems. Actual control law can be obtained from the last step[9]. So, the backstepping technique is a flexible approach in designing controllers for nonlinear systems. Backstepping method was used to control spacecraft formation flying in some researches[1,10-12]. Adaptive backstepping controller was designed for satellite formation with mass uncertainty and thruster error in some studies [11,12]. In another study, Kristiansen et al. developed an integrator backstepping approach based on nonlinear model of relative motion[12].

In the present work, sliding mode control and backstepping technique are combined to design a robust backstepping sliding mode controller (BSMC) for spacecraft formation control on elliptical orbits. After applying the backstepping method to incorporate the tracking errors and Lyapunov functions, a sliding mode controller is developed to guarantee the Lyapunov stability, handling all system nonlinearities, robustness against uncertainties and disturbances and tracking the desired formation. In this paper, it is assumed that the spacecraft move in low Earth orbits and are subjected to the perturbing effects of J_2 and atmospheric drag. The performance of the proposed controller in tracking the desired trajectory is compared to a conventional sliding mode controller.

SPACECRAFT RELATIVE MOTION DYNAMICS

In this section, dynamic model of relative motion for SFF is specified. It is assumed that each spacecraft is a point mass. As shown in Figure 1, $C_1 = \{X, Y, Z\}$ is the inertial coordinate system and $\boldsymbol{r}_l$ and $\boldsymbol{r}_f$ are the position vectors of leader and follower spacecraft, respectively. The coordinate system $C_2 = \{x, y, z\}$ is a moving frame located on the leader's center of mass. Herein, y axis is along the direction of $\boldsymbol{r}_l(t)$, x is along the direction of leader velocity vector and normal to y, and z axis completes the right-handed C_2 coordinate frame.

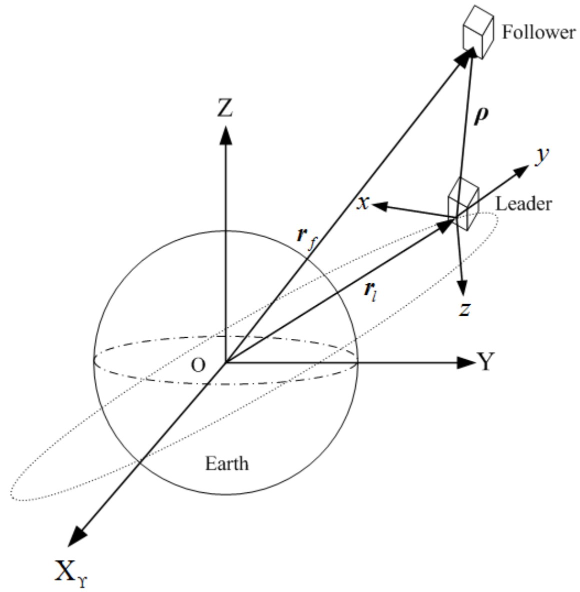

Figure 1. Inertial and Moving Coordinate Systems

The dynamics of the leader and follower in the inertial reference frame can be written as

$$\ddot{\boldsymbol{r}}_l + \frac{\mu}{r_l^3}\boldsymbol{r}_l = \boldsymbol{d}_l + \boldsymbol{u}_l \tag{1}$$

$$\ddot{\boldsymbol{r}}_f + \frac{\mu}{r_f^3}\boldsymbol{r}_f = \boldsymbol{d}_f + \boldsymbol{u}_f \tag{2}$$

where $r = \|\boldsymbol{r}\|$, $\mu = 398\,600\ \text{km}^3/\text{s}^2$ is the constant of the Earth gravity, $\boldsymbol{u}$ is control input vector and $\boldsymbol{d}$ is the vector of external disturbance. It is supposed that the follower spacecraft should be controlled; the leader is subjected to perturbations, and it moves in an uncontrolled ballistic trajectory; therefore $\boldsymbol{u}_l = 0$. Position vector of the follower relative to the leader is defined as $\boldsymbol{\rho} = \boldsymbol{r}_f - \boldsymbol{r}_l = [x \quad y \quad z]^T$. The nonlinear relative dynamics of the follower with respect to the moving frame is described as[13]

$$\ddot{\boldsymbol{\rho}} + \boldsymbol{C}(\omega_l)\dot{\boldsymbol{\rho}} + \boldsymbol{F}(\boldsymbol{\rho}, \omega_l, \dot{\omega}_l, \boldsymbol{r}_l, N, n) = \boldsymbol{D} + \boldsymbol{u}_f \tag{3}$$

where

$$\boldsymbol{D} = \boldsymbol{d}_f - \boldsymbol{d}_l \tag{4}$$

$\boldsymbol{D}$ is the differential perturbation subjecting to the formation and ,

$$\boldsymbol{C}(\omega_l) = 2\omega_l \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{F}(\boldsymbol{\rho}, \omega_l, \dot{\omega}_l, \boldsymbol{r}_l, N, n) = \begin{bmatrix} (N^2 - \omega_l^2)x + \dot{\omega}_l y \\ (N^2 - \omega_l^2)y + (N^2 - n^2)r_l - \dot{\omega}_l x \\ N^2 z \end{bmatrix} \tag{5}$$

and

$$\omega_l = \dot{\theta} = \frac{\sqrt{\mu a(1-e^2)}}{r_l^2}, \quad \dot{\omega}_l = \frac{-2\mu e \sin\theta}{r_l^3}, \quad n = \sqrt{\frac{\mu}{r_l^3}}$$
$$N = \sqrt{\frac{\mu}{\left[x^2 + (r_l + y)^2 + z^2\right]^{3/2}}}, \quad r_l = \frac{a(1-e^2)}{1+e\cos\theta} \tag{6}$$

in which ω_l, a, e, θ are the angular velocity, the orbital semi-major axis, the eccentricity and the true anomaly of the leader spacecraft, respectively.

CONTROLLER DESIGN

Sliding Mode Control

Sliding mode controller as a robust controller is an appropriate technique for nonlinear systems control. It provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling inaccuracies[14].

Because J_2 disturbance and atmospheric drag are dominant perturbations in low-Earth orbits, we consider them as external disturbances in this study. They have bounded values based on spacecraft altitude, so for D_i in (6) we have

$$|D_i| \le q_i \quad i = 1,2,3 \tag{7}$$

where the positive constant q_i is considered as[15,16]

$$q_i = 10^{-7}\ \mathrm{km/s^2} \tag{8}$$

The sliding surface is chosen as

$$\boldsymbol{s} = \dot{\boldsymbol{e}} + \boldsymbol{\lambda}\boldsymbol{e} \tag{9}$$

where $\boldsymbol{e} = \boldsymbol{\rho} - \boldsymbol{\rho}_d$ and $\dot{\boldsymbol{e}} = \dot{\boldsymbol{\rho}} - \dot{\boldsymbol{\rho}}_d$. $\boldsymbol{\rho}_d, \dot{\boldsymbol{\rho}}_d \in R^3$ are the desired relative position and velocity of the follower. $\boldsymbol{s} = [s_1, s_2, s_3]^T$ and $\boldsymbol{\lambda} = diag(\lambda_1, \lambda_2, \lambda_3)$ are sliding surfaces and slopes of sliding surfaces, respectively.

According to the sliding mode control methodology[14], the control input law is determined as

$$\boldsymbol{u}_f = \boldsymbol{C}(\cdot)\dot{\boldsymbol{\rho}} + \boldsymbol{F}(\cdot) + \ddot{\boldsymbol{\rho}}_d - \boldsymbol{\lambda}\dot{\boldsymbol{e}} - \boldsymbol{k}\ \mathrm{sgn}(\boldsymbol{s}) \tag{10}$$

where $\boldsymbol{k} = diag(k_1, k_2, k_3)$ is the gain matrix determined based on the amount of uncertainties, disturbance and reaching time to sliding surface. Sign function vector $\mathrm{sgn}(\boldsymbol{s})$ is a column matrix of sign functions

$$\mathrm{sgn}(\boldsymbol{s}) = \left[\mathrm{sgn}(s_1), \mathrm{sgn}(s_2), \mathrm{sgn}(s_3)\right]^T \tag{11}$$

To guarantee stability, Lyapunov second method is used; $V = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{s}$ is considered as Lyapunov function and k_i is determined in such a way that $\dot{V} < 0$. Thus by choosing $k_i > q_i$, the closed-loop system will be globally asymptotically stable. In practice, sign functions in Equation (10) can be replaced with continuous saturation functions to eliminate chattering phenomenon, then the controller law becomes

$$\boldsymbol{u}_f = \boldsymbol{C}(\cdot)\dot{\boldsymbol{\rho}} + \boldsymbol{F}(\cdot) + \ddot{\boldsymbol{\rho}}_d - \boldsymbol{\lambda}\dot{\boldsymbol{e}} - \boldsymbol{k}\ sat(\boldsymbol{s}, \varphi) \tag{12}$$

where $sat(\boldsymbol{s},\varphi)=\left[sat(s_1,\varphi),sat(s_2,\varphi),sat(s_3,\varphi)\right]^T$ and saturation function $sat(s_i,\varphi)$ is

$$sat\,(s_i\,,\varphi)=s_i\Big/\left(\left|s_i\right|+\varphi\right) \tag{13}$$

where φ is a positive scalar constant.

Backstepping Sliding Mode Control

In order to utilize the benefits offered by the sliding mode and backstepping controller, these two techniques are combined to develop a backstepping sliding mode controller for spacecraft formation. First, state-space form of the nonlinear dynamic (3) is considered $\left(\boldsymbol{X}_1=\boldsymbol{\rho},\quad \boldsymbol{X}_2=\dot{\boldsymbol{X}}_1\right)$:

$$\begin{cases} \dot{\boldsymbol{X}}_1=\boldsymbol{X}_2 \\ \dot{\boldsymbol{X}}_2=-\boldsymbol{C}\,(\cdot)\boldsymbol{X}_2-\boldsymbol{F}\,(\cdot)+\boldsymbol{D}+\boldsymbol{u}_f \end{cases} \tag{14}$$

Tracking error can be defined as

$$\boldsymbol{Z}_1=\boldsymbol{X}_1-\boldsymbol{X}_d \tag{15}$$

so $\boldsymbol{X}_2=\boldsymbol{\alpha}$ is considered as virtual control for first subsyestem. It can be determined from the first Lyapunov function as follows

$$V_1=\frac{\gamma}{2}\boldsymbol{Z}_1^T\boldsymbol{Z}_1 \tag{16}$$

where $\gamma>0$; accordingly $\dot{V}_1$ is

$$\dot{V}_1=\gamma\boldsymbol{Z}_1^T\dot{\boldsymbol{Z}}_1=\gamma\boldsymbol{Z}_1^T\left[\boldsymbol{\alpha}-\dot{\boldsymbol{X}}_d\right] \tag{17}$$

Virtual control input is selected as

$$\boldsymbol{\alpha}=\dot{\boldsymbol{X}}_d-\boldsymbol{K}_1\boldsymbol{Z}_1 \tag{18}$$

where K_1 is a positive diagonal matrix. So $\dot{V}_1$ becomes

$$\dot{V}_1=-\gamma\boldsymbol{Z}_1^T\boldsymbol{K}_1\boldsymbol{Z}_1<0 \tag{19}$$

which means that the first subsystem is asymptitically stable. The second step of the controller design follows by defining the new variable $\boldsymbol{Z}_2=\boldsymbol{X}_2-\boldsymbol{\alpha}$. So we have

$$\begin{cases} \boldsymbol{Z}_2=\boldsymbol{X}_2-\dot{\boldsymbol{X}}_d+\boldsymbol{K}_1\boldsymbol{Z}_1=\dot{\boldsymbol{Z}}_1+\boldsymbol{K}_1\boldsymbol{Z}_1 \\ \dot{\boldsymbol{Z}}_2=-\boldsymbol{C}\,(\cdot)\boldsymbol{X}_2-\boldsymbol{F}\,(\cdot)+\boldsymbol{D}+\boldsymbol{u}_f-\dot{\boldsymbol{\alpha}} \end{cases} \quad (20$$

Sliding surface is defined as

$$\boldsymbol{s}=\boldsymbol{Z}_2+\boldsymbol{\eta}\boldsymbol{Z}_1 \tag{21}$$

where $\boldsymbol{\eta}=diag\,(\eta_1,\eta_2,\eta_3)$ are the slopes of sliding surfaces. Second Lyapunov function is considered as follows

$$V_2=V_1+\frac{1}{2}\boldsymbol{s}^T\boldsymbol{s} \tag{22}$$

Differentiating (22) gives

$$\dot{V}_2 = \gamma \boldsymbol{Z}_1^T \dot{\boldsymbol{Z}}_1 + \boldsymbol{s}^T \dot{\boldsymbol{s}} = \gamma \boldsymbol{Z}_1^T \left[\boldsymbol{Z}_2 - \boldsymbol{K}_1 \boldsymbol{Z}_1\right] + \boldsymbol{s}^T \left[\dot{\boldsymbol{Z}}_2 + \boldsymbol{\eta} \dot{\boldsymbol{Z}}_1\right]$$
$$= \gamma \boldsymbol{Z}_1^T \boldsymbol{Z}_2 - \gamma \boldsymbol{Z}_1^T \boldsymbol{K}_1 \boldsymbol{Z}_1 + \boldsymbol{s}^T \left[-\boldsymbol{C}(\cdot)\boldsymbol{X}_2 - \boldsymbol{F}(\cdot) + \boldsymbol{D} + \boldsymbol{u}_f - \dot{\boldsymbol{\alpha}} + \boldsymbol{\eta}\left(\boldsymbol{Z}_2 - \boldsymbol{K}_1 \boldsymbol{Z}_1\right)\right] \tag{23}$$

Backstepping sliding mode control law is designed as

$$\boldsymbol{u}_f = \boldsymbol{C}(\cdot)\boldsymbol{X}_2 + \boldsymbol{F}(\cdot) + \dot{\boldsymbol{\alpha}} - \boldsymbol{\eta}\left(\boldsymbol{Z}_2 - \boldsymbol{K}_1 \boldsymbol{Z}_1\right) - \boldsymbol{K}_2 \boldsymbol{s} - \boldsymbol{K}_3 \operatorname{sgn}(\boldsymbol{s}) \tag{24}$$

where $\boldsymbol{K}_2$ and $\boldsymbol{K}_3$ are positive diagonal matrices. Thus Equation (23) is written as

$$\dot{V}_2 = \gamma \boldsymbol{Z}_1^T \boldsymbol{Z}_2 - \gamma \boldsymbol{Z}_1^T \boldsymbol{K}_1 \boldsymbol{Z}_1 - \boldsymbol{s}^T \boldsymbol{K}_2 \boldsymbol{s} + \boldsymbol{s}^T \left[\boldsymbol{D} - \boldsymbol{K}_3 \operatorname{sgn}(\boldsymbol{s})\right] \tag{25}$$

We know that $|D_i| \le q_i$, so we have

$$\boldsymbol{s}^T \left[\boldsymbol{D} - \boldsymbol{K}_3 \operatorname{sgn}(\boldsymbol{s})\right] \le \sum_{i=1}^{3} |s_i| (q_i - K_{3i}) \tag{26}$$

thus by choosing $K_{3i} > q_i$, the time derivative of Lyapunov function is simplified to

$$\dot{V}_2 \le \gamma \boldsymbol{Z}_1^T \boldsymbol{Z}_2 - \gamma \boldsymbol{Z}_1^T \boldsymbol{K}_1 \boldsymbol{Z}_1 - \boldsymbol{s}^T \boldsymbol{K}_2 \boldsymbol{s} \tag{27}$$

Note that Equation (27) can be rewritten as

$$\dot{V}_2 \le -\boldsymbol{Z}^T \boldsymbol{Q} \boldsymbol{Z} \tag{28}$$

where

$$\boldsymbol{Q} = \begin{bmatrix} \gamma \boldsymbol{K}_1 + \boldsymbol{\eta}^T \boldsymbol{K}_2 \boldsymbol{\eta} & \boldsymbol{\eta}^T \boldsymbol{K}_2 - \dfrac{\gamma}{2}\boldsymbol{I} \\ \boldsymbol{K}_2 \boldsymbol{\eta} - \dfrac{\gamma}{2}\boldsymbol{I} & \boldsymbol{K}_2 \end{bmatrix} \tag{29}$$

and $\boldsymbol{Z}^T = \left[\boldsymbol{Z}_1 \quad \boldsymbol{Z}_2\right]$. According to the Schur's lemma[17], if $\boldsymbol{K}_2\left(\boldsymbol{K}_1 + \boldsymbol{\eta}\right) - (\gamma/4)\boldsymbol{I} > 0$ matrix $\boldsymbol{Q}$ is positive definite; Therefore $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ will converge to zero, and the asymptotical stability of the proposed backstepping sliding mode control system is guaranteed.

In order to overcome the chattering phenomenon, saturation function (13) is considered instead of sign function in the proposed control input (24).

SIMULATION RESULTS AND DISCUSSION

The nonlinear model of spacecraft formation (3) is used for simulation purposes. For control part, it is assumed that $\boldsymbol{\rho}$ and $\dot{\boldsymbol{\rho}}$ are measurable and available. The perturbation due to the Earth's oblateness with respect to the inertial frame is given as[15]

$$\boldsymbol{D}_{J_2} = \frac{\mu J_2 R_e^2}{2} \begin{bmatrix} \dfrac{15Z^2X}{\|\boldsymbol{r}\|^7} - \dfrac{3X}{\|\boldsymbol{r}\|^5} \\ \dfrac{15Z^2Y}{\|\boldsymbol{r}\|^7} - \dfrac{3Y}{\|\boldsymbol{r}\|^5} \\ \dfrac{15Z^3}{\|\boldsymbol{r}\|^7} - \dfrac{9Z}{\|\boldsymbol{r}\|^5} \end{bmatrix} \tag{30}$$

where $J_2 = 0.0010826$, $R_e = 6378.137$ km is the mean equatorial radius of the Earth. This perturbation can be transformed to the moving frame by using coordinate transformation. Atmospheric drag is also given as[16]

$$\boldsymbol{D}_{drag} = -\frac{1}{2}\frac{C_D S}{m}\sigma v^2 \hat{\mathbf{v}} \tag{31}$$

where m is the mass of spacecraft, C_D is the drag coefficient, S is the effective surface, σ is the local atmosphere density, v is the relative velocity of the spacecraft with respect to the atmosphere and $\hat{\mathbf{v}}$ is the related unit vector. Constant parameters are assumed as follows:

$$m = 100\text{kg} \quad C_D = 2 \quad S = 0.5\text{m}^2$$

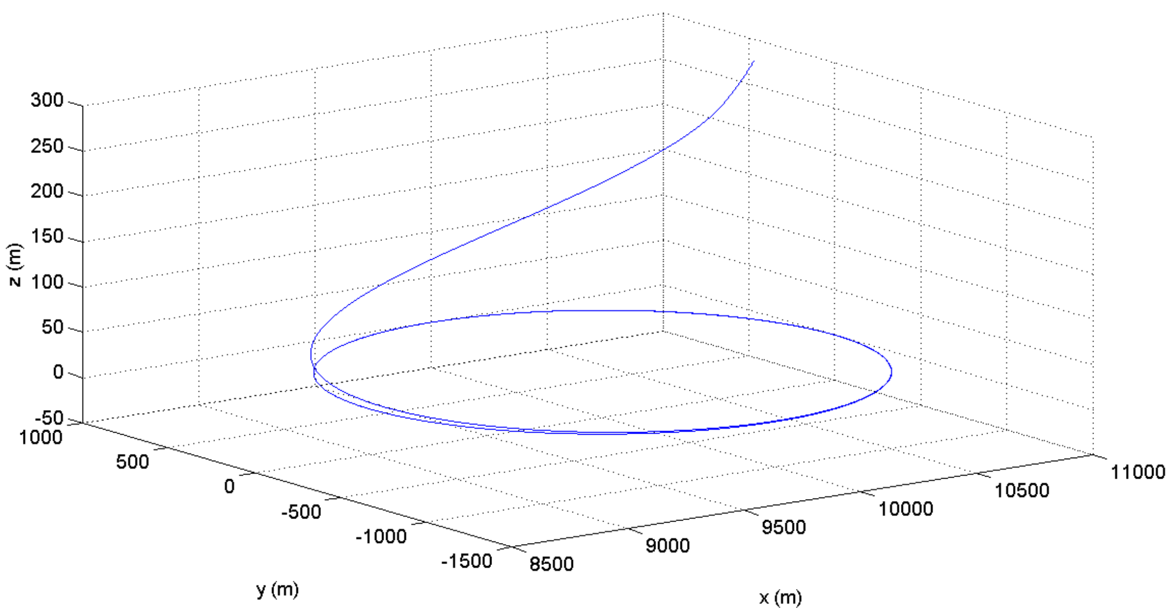

Figure 2. Three-Dimensional Trajectory of the Follower Using BSMC

The most significant density variation is due to the height above the surface. Locally, this variation might be accounted for by the barometric formula[16]

$$\sigma(h) = \sigma_0 \exp\left[-\frac{h-h_0}{H}\right] \tag{32}$$

where σ_0 is the density at the reference height h_0, and H is the scaling height at h_0. The reference values are selected as[18]

$$h_0 = 600\text{km} \quad \sigma_0 = 1.454\times10^{-13}\,\text{kg/m}^3 \quad H = 71.835\text{km}$$

The initial orbital elements of the leader are supposed as:

$$a = 7378.137\text{km} \quad e = 0.1 \quad i = 30^\circ \quad \omega = 45^\circ \quad \Omega = \theta = 0^\circ$$

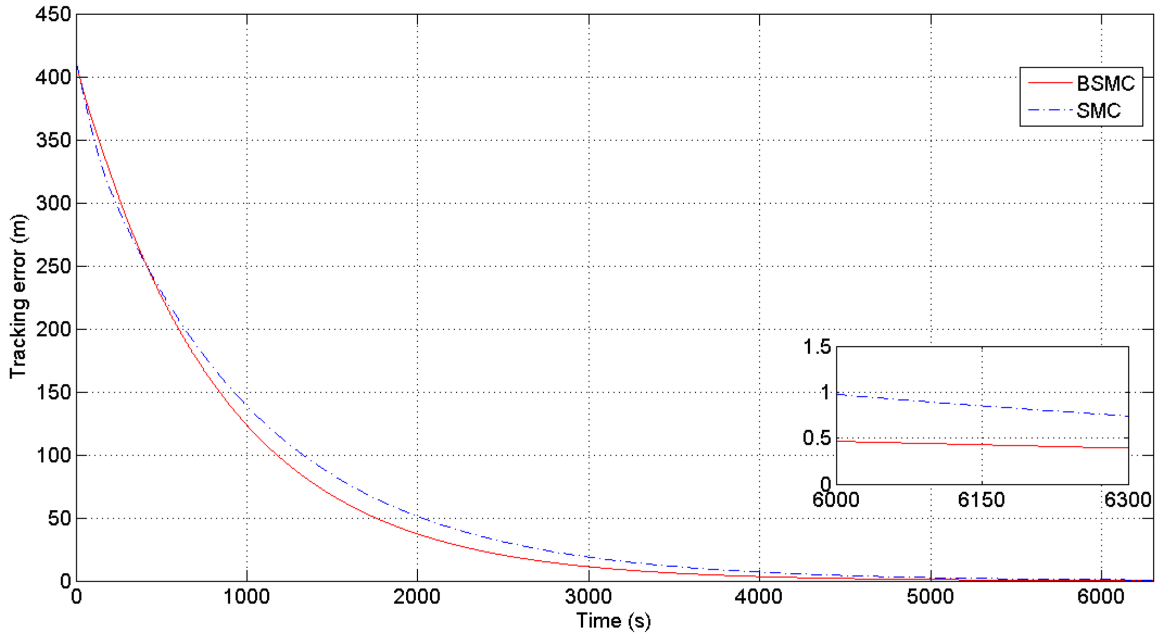

Figure 3. Total Tracking Error

The desired relative trajectory of the follower is a circular formation with a radius of 1 km in x-y plane[19]. The center of the desired formation is located on (10000,0,0)m. The period of the follower movement entirely lies on the leader spacecraft angular velocity around the Earth and is obtained as $T \approx 6300\text{s}$. The initial relative errors of the follower in the moving frame are chosen as

$$\begin{cases} (e_x \quad e_y \quad e_z) = (-200 \quad 200 \quad 300)(\text{m}) \\ (\dot{e}_x \quad \dot{e}_y \quad \dot{e}_z) = (0 \quad -1.22 \quad 0)(\text{m}/\text{s}) \end{cases} \tag{33}$$

The parameters of the both controllers are considered below

$$\boldsymbol{\lambda} = 0.001 \times \text{diag}(1,1,1), \quad \boldsymbol{k} = (4\times 10^{-6}) \times \text{diag}(1,1,1)$$
$$\boldsymbol{\eta} = (6\times 10^{-4}) \times \text{diag}(1,1,1), \quad \boldsymbol{K}_1 = (6\times 10^{-4}) \times \text{diag}(1,1,1), \quad \boldsymbol{K}_2 = 0.003 \times \text{diag}(1,1,1), \quad \boldsymbol{K}_3 = (10^{-6}) \times \text{diag}(1,1,1)$$

and $\varphi = 10^{-5}$ for both controllers.

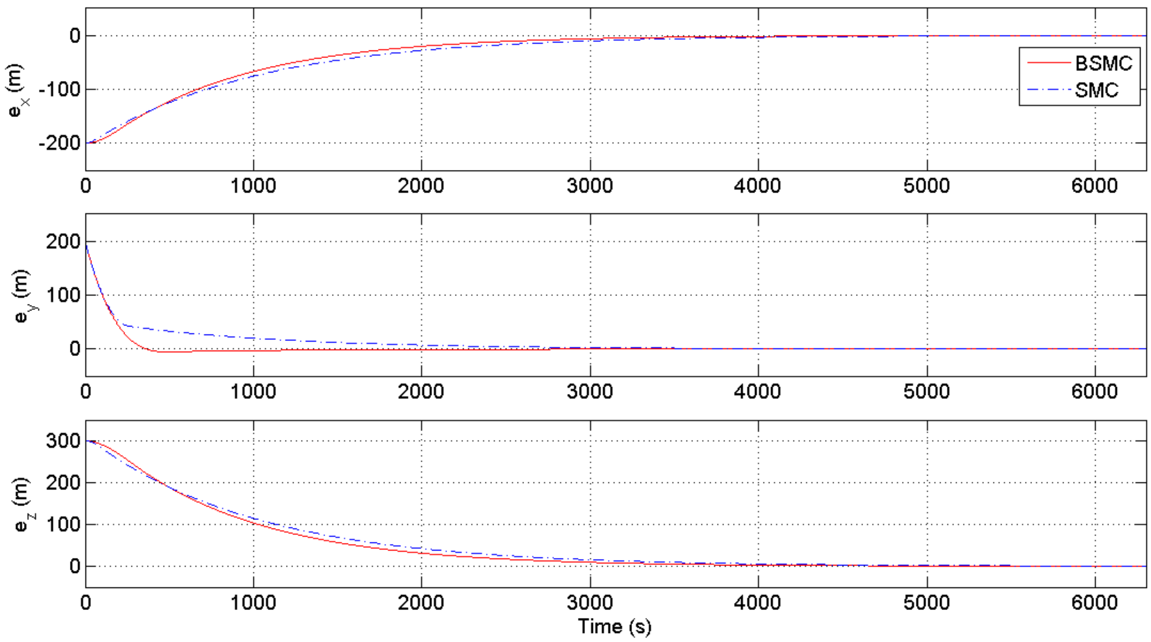

Figure4. Tracking Error along x-, y- and z- Axis

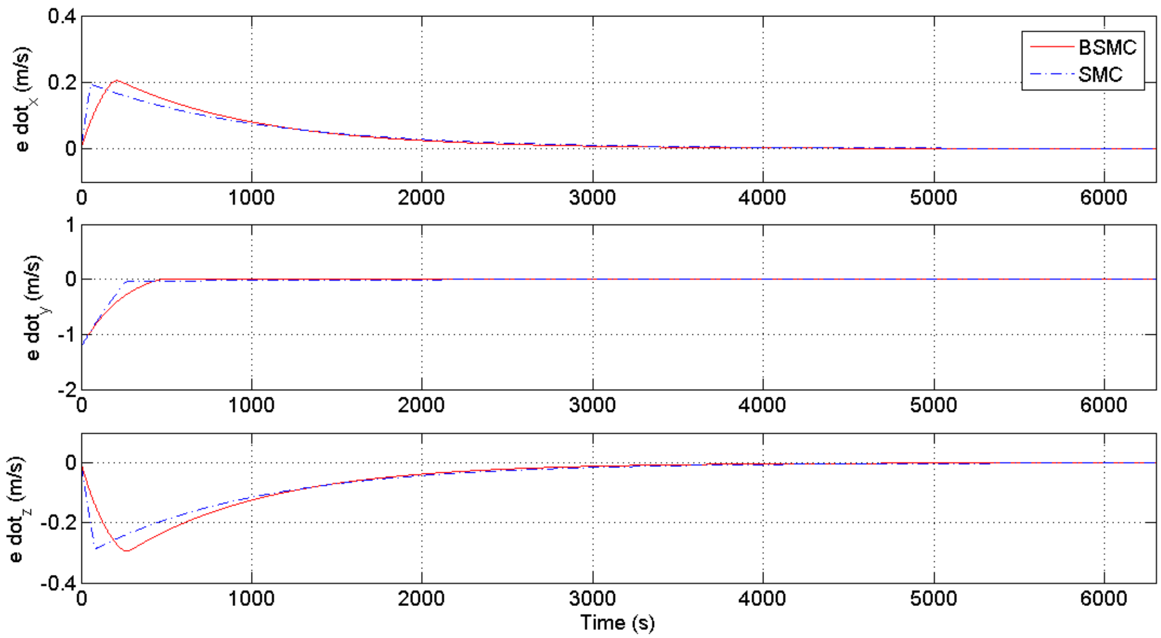

Figure5. Relative Velocity Error along x-, y- and z- Axis

According to the selected gains, stability condition for both controllers is fulfilled. Figure 2 shows a three-dimensional view of the follower motion relative to the leader using the proposed backstepping sliding mode controller input. System's response in reducing the tracking errors during one period of movement using both controllers have been presented in Figure 3 and Figure 4. Relative velocity error has been also demonstrated in Figure 5.

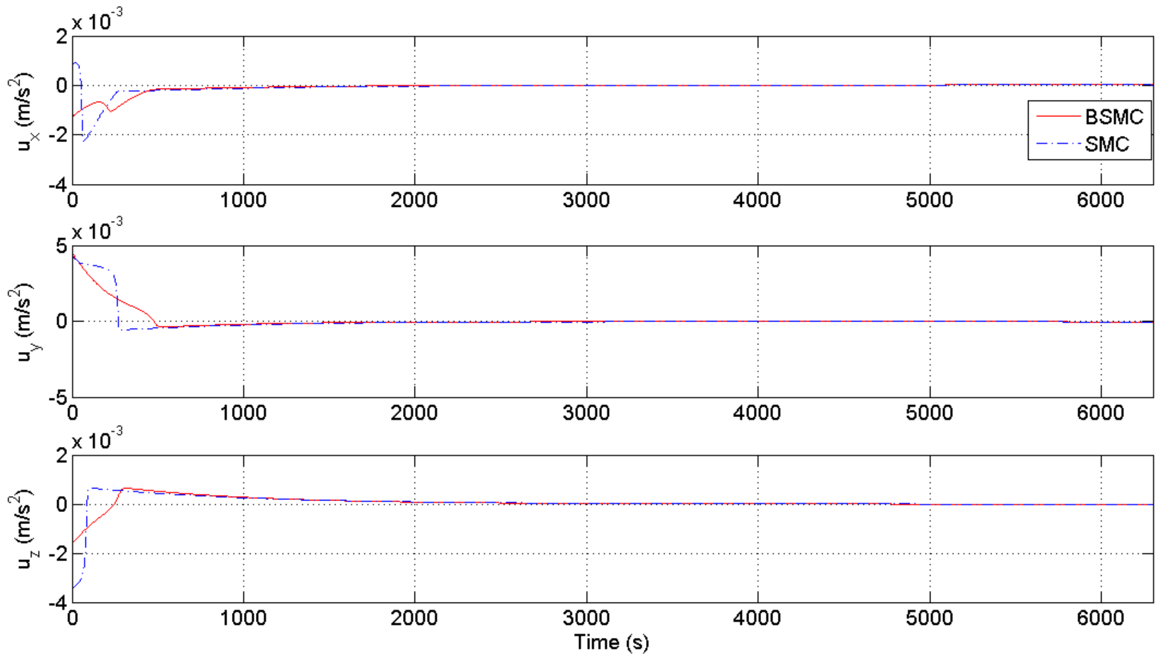

Figure 6. Control Inputs of the Follower Using BSMC and SMC

As shown in the figures, the backstepping sliding mode controller has more accurate response in tracking the desired formation. Figure 6 shows control inputs, and fuel cost has been demonstrated in Figure 7. Fuel cost can be obtained as

$$\Delta V = \int_0^t \left(|u_x| + |u_y| + |u_z| \right) \mathrm{dt} \tag{34}$$

Table 1. Comparison of Both Controllers at the End of One Orbit

Controller	$\|e\|(m)$	$\Delta V(m/s)$
BSMC	0.39	2.55
SMC	0.74	2.98

The performance of both controllers at the end of one orbit have been compared in Table 1, in which $\|e\|$ is total tracking error and ΔV is fuel cost. Based on the results, the proposed backstepping sliding mode controller has a superior performance in tracking the desired trajectory and also has lower fuel cost compared with the conventional sliding mode controller.

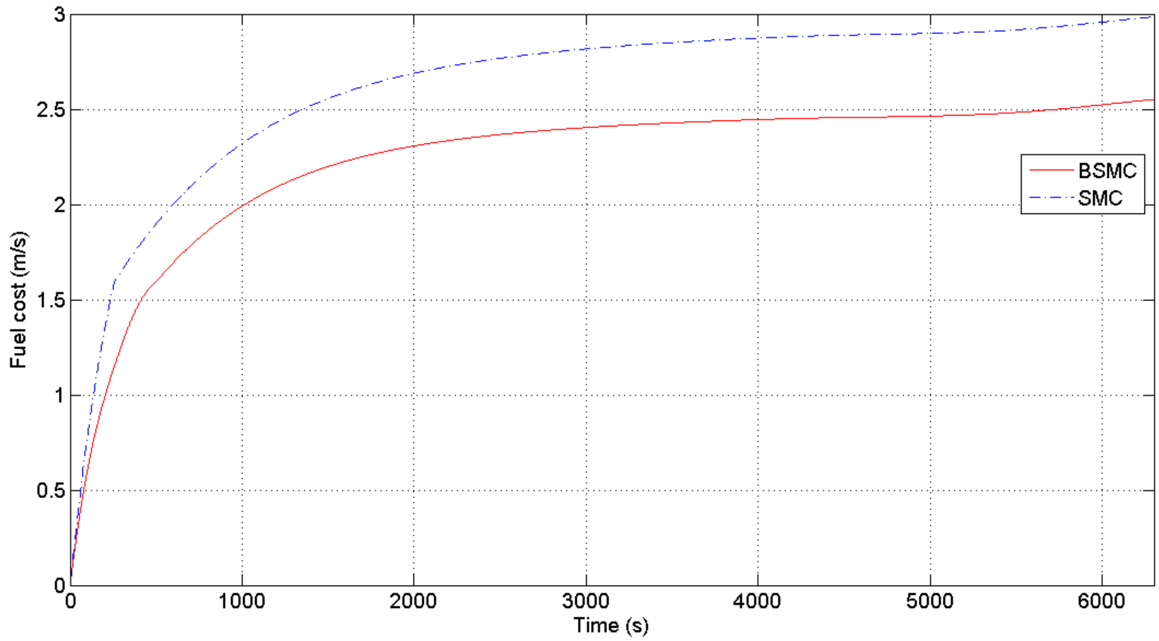

Figure7. Fuel Cost of the Follower Using BSMC and SMC

CONCLUSION

In this study, a backstepping sliding mode controller was designed for spacecraft formation control on elliptical orbits. The controller design was based on the nonlinear model of spacecraft formation while perturbation of J_2 and atmospheric drag were considered as external disturbances. Using Lyapunov second theory, the stability of the closed-loop system was guaranteed. In comparison the performance of the proposed backstepping sliding mode controller to a conventional sliding mode controller, simulation results confirmed more efficient and superior performance of the proposed controller in tracking the desired formation and fuel cost.

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