# Fuzzy Non-monotonic Logic Using Two Membership Functions Known and Unknown

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***Abstract*—John McCarthy proposed non-monotonic reasoning which is undecidable and has nolution. An undecided problem has no solution. Fuzzy logic will made undecidable problem into decidable. In this paper, fFuzzy non-monotonic reasoning is studied with a twofold fuzzy logic to made undecidable problem in to decidable. Fuzzy truth maintenance system (FTMS) is studied for computation of fuzzy non-monotonic reasoning.**

***Keywords*—non-monotonic reasoning, fuzzy Sets, twofold fuzzy sets, fuzzy non-monotonic reasoning, FTMS,incomplete knowledge**

## I. Introduction

Sometimes Artificial Intelligence(AI) has to deal with incomplete knowledge. If knowledge base is incomplete then the inference is also incomplete. If some knowledge is added to the system than the inference is changes in non-monotonic reasoning. In non-monotonic reasoning, if additional information is added, the reasoning will be changed or jumping conclusion [4].

X is bird Λ x has wings Λ x is known to fly → x can fly
Suppose ,
x is bird Λ x has wings Λ x is unknown to fly → x can fly
or
x is bird Λ x has wings Λ x is unknown to fly → x can't fly
For example,
Ozzie is bird Λ Ozzie has wings Λ Ozzie x is known to fly → Ozzie can fly
Ozzie is bird Λ Ozzie has wings Λ Ozzie x is unknown to fly → Ozzie can't fly
Ozzie is bird Λ Ozzie has wings Λ Ozzie is unknown to fly → Ozzie can fly

Consider the formula,

$\forall x P(x) \Lambda \forall x Q(x) \Lambda \forall x R(x) \rightarrow \forall x S(x)$

The monotonic logic is given by

$\forall x$ bird(x)Λ$\forall x$ Wings(x)Λ$\forall x$ known-to-fly(x)→$\forall x$ fly(x)

Inference S is changed if R is changed. The non-monotonic logic is given by

$\forall x$ bird(x)Λ$\forall x$ Wings(x)Λ$\forall x$ unknown-to-fly(x)→$\forall x$ can-fly(x) or $\forall x$ can't-fly(x)

The conclusion will be changed if added some knowledge in non-monotonic logic. These problems fall under undecided. The undecided problems have no solution. Fuzzy logic will made undecidable problems in to decidable

There are many theories [1] to deal with incomplete information like Probability, Dempster- Shaffer theory, Possibility, Plausibility etc. Zadeh [14] fuzzy logic is based on belief rather than probable (likelihood). The fuzzy logic made imprecise information in to precise.

Zadeh fuzzy logic is defined with single membership function.

Fuzzy logic with two membership functions will give more information

Two fold fuzzy logic P=(A, B) for the proposition of the type "x is P".

P may be considered as
P={belief, disbelief}, {True, false}, {unknown, known}, {belief, disbelief} etc.

$\mu_P(x) \Lambda \mu_Q(x) \rightarrow \mu_S(x)$

$\mu_P(x)^{(unknown,\ known)} \Lambda \mu_Q(x)^{(unknown\text{-},\ known)} \rightarrow \mu_S(x)^{(unknown,\ known)}$

where P,Q and S are twofold fuzzy set known, known}.

$\mu_{bird}(x) \Lambda \mu_{wings}(x) \rightarrow \mu_{fly}(x)$

$\mu_{bird}(x)^{(unknown,known)} \Lambda \mu_{wings}(x)^{(unknown\text{-},known)} \rightarrow \mu_{fly}(x)^{(unknown,\ known)}$

The conflict of the incomplete information may be defend by fuzzy certainty factor(FCF).

FCF =unknown- known)

$\mu_{bird}^{FCF}(x) \Lambda \mu_{wings}^{FCF}(x) \rightarrow \mu_{fly}^{FCF}(x)$

Where known and unknown are the fuzzy membership functions.

The fuzzy non-monotonic reasoning will bring uncertain knowledge in to certain knowledge.

$\mu_P(x)^{(unknown,\ known)} \Lambda \mu_Q(x)^{(unknown\text{-},known)} \rightarrow \mu_S(x)$

$S = \mu_S^{FCF}(x) = 1 \quad \mu_S^{FCF}(x) \leq \alpha,$
$\qquad 0 \quad \mu_S^{FCF}(x) > \alpha$

$\mu_{bird}(x)^{(unknown,\ known)} \Lambda \mu_{wings}(x)^{(unknown\text{-},known)} \rightarrow \mu_{fly}(x)$

$fly = \mu_{fly}^{FCF}(x) = 1 \quad \mu_{fly}^{FCF}(x) \leq \alpha,$
$\qquad 0 \quad \mu_{fly}^{FCF}(x) > \alpha$

Fly for 1 and can't fly for 0

## II. Fuzzy Logic

The possibility set may be defined for the proposition of the type "x is P" as

$\pi_P(x) \rightarrow [0,1]$

$\pi_P(x) = \max\{ \mu_P(x_i) \},\ x \epsilon X$

$\mu_P(x) = \mu_P(x_1)/x_1 + \mu_P(x_2)/x_2 + \ldots + \mu_P(x_n)/x_n$

$\mu_{bird}(x) = \mu_{bird}(x_1)/x_1 + \mu_{bird}(x_2)/x_2 + \ldots + \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x) = \mu_{bird}(x_1)/x_1 + \mu_{bird}(x_2)/x_2 + \ldots + \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)$= 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos

Let P and Q be the fuzzy sets, and the operations on fuzzy sets are given below [13]

$PVQ = \max(\mu_P(x), \mu_Q(x)\}$ Disjunction

$P\Lambda Q = \min(\mu_P(x), \mu_Q(x)\}$ Conjunction

$P'=1-\mu_P(x)$ Negation

$PxQ=\min\{\mu_P(x), \mu_Q(x)\}$ Relation

$P \text{ o } Q==\min\{\mu_P(x), \mu_Q(x, x)\}$ Composition

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

$\mu_{very\ P}(x)=\mu_P(x)^2$ Concentration

$\mu_{more\ or\ less\ P}(x)=\mu_P(x)^{0.5}$ Diffusion

**Quasi-fuzzy set**

A quasi-fuzzy set is defined for the proposition " x is P" as

$\mu_P(x) \rightarrow (0, 1)$

## III. FUZZY CONDITIONAL INFERENCE

The fuzzy rules are of the form "if <Precedent Part> then <Consequent Part>"

if x is P the n x is Q.

or

. if x is $P_1$ and x is $P_2$ …. x is $P_n$ then x is Q

The Zadeh [13] fuzzy conditionali nference s given by

if x is $P_1$ and x is $P_2$ …. x is $P_n$ then x is Q =

$\min 1, (1-\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x)) + \mu_Q(x)\}$ (2.1)

The Mamdani [8] fuzzy conditional inference s given by

if x is $P_1$ and x is $P_2$ …. x is $P_n$ then x is Q =

$\min\{\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x), \mu_Q(x)\}$ (2.2)

Reddy [10] The fuzzy conditional inference "Consequent Part" is derived from "Precedent Part" .

if x is $P_1$ and x is $P_2$ …. x is $P_n$ then x is Q =

ix is $P_1$ and x is $P_2$ …. x is $P_n$

Fuzzy conditional inference is given by using mamdani fuzzy conditional inference

$= \min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$ (2.3)

.

For instance, x is bird Λ x has wings → x can fly

x can fly= x is bird Λ x has wings

**Quasi-fuzzy set**

A quasi-fuzzy set is defined for the proposition " x is P" as

$\mu_P(x) \rightarrow (0, 1)$

$\mu_{fly}(x)^{(can,\ can't)} \rightarrow (0,1)$

## IV. THE TWO FOLD FUZZY LOGIC

Zadeh [13] Proposed fuzzy set with single membership function. The two fold fuzzy set [12] will give more evidence than single membership function.

The fuzzy non-monotonic set may defined with two fold membership function using unknown and known

**Definition**: Given some Universe of discourse X, the proposition " x is P" is defined as its two fold fuzzy membership function as

$\mu_P(x) = \{\mu_P^{True}(x), \mu_{False}(x)\}$

Interpreting "truth is knkown but false is known", the twofold fuzzy set is given by

$P = \{\mu_P^{unknown}(x), \mu_P^{known}(x)\}$

Where P is Generalized fuzzy set and x Є X ,

$0 <= \mu_P^{unknown}(x) <=1$ and, $0 <= \mu_P^{known}(x) <=1$

$P = \{\mu_P^{unknown}(x_1)/x_1 + \ldots + \mu_P^{unknown}(x_n)/x_n,$
$\mu_P^{known}(x_1)/x_1 + \ldots + \mu_P^{unknown}(x_n)/x_n,\ x_i \in X$, "+" is union

For example ' x will fly" , fly may be given as

Suppose P and Q is fuzzy non-monotonic sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

**Negation**

$P' = \{1-\mu_P^{unknown}(x), 1-\mu_P^{known}(x)\ \}/x$

**Disjunction**

$PVQ=\{\max(\mu_P^{known}(x), \mu_P^{known}(y)), \max(\mu_Q^{unknown}(x), \mu_Q^{unknown}(x))\}$

**Conjunction**

$P\Lambda Q=\{\min(\mu_P^{known}(x), \mu_P^{known}(y)), \min(\mu_Q^{unknown}(x), \mu_Q^{unknown}(x))\}$

**Implication**

Zadeh fuzzy conditional inference

$P \rightarrow Q = \{\min(1, 1-\mu_P^{known}(x)+\mu_Q^{known}(x), \min(1, 1-\mu_P^{known}(x)+\mu_Q^{known}(x)\}$

Mamdani fuzzy conditional inference

$P \rightarrow Q = \{\min(\mu_P^{unknown}(x), \mu_Q^{unknown}(y), \min(\mu_P^{known}(x), \mu_Q^{known}(x)\}$

Reddy fuzzy conditional inference

$P \rightarrow Q = \{\min(\mu_P^{unknown}(x), \mu_P^{known}(y)\}(x,x)$

**Composition**

$P \text{ o } R = \{\min_x(\mu_P^{unknown}(x), \mu_P^{unknown}(x)), \min_x(\mu_R^{known}(x), \mu_R^{known}(x))\}$

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

**Concentration**

"x is very P

$\mu_{very\ P}(x) = \{\mu_{very\ P}^{unknown}(x)^2, \mu_{very\ P}^{known}(x)\mu_P(x)^2\}$

**Diffusion**

"x is more or less P"

$\mu_{more\ or\ less\ P}(x) = (\mu_{more\ or\ less}^{unknown}(x)^{1/2}, \mu_{more\ or\ less}^{known}(x)\mu_P(x)^{0.5}$

The fuzzy certainty factor (FCF) is defined by fuzziness instead of probability for the fuzzy preposition of the type " x is A"

FCF[x, A]=MB[x, A]-MD[x,A],

The FCF is the difference between "unknown" and "known" and will eliminate conflict between "unknown" and "known" and, made as single membership function

$\mu_A^{FCF}(x) = \mu_A^{unknown}(x) - \mu_A^{known}(x)$

**Quasi-fuzzy set**

A quasi-fuzzy set is defined for the proposition " x is P" as

$\mu_P(x) \rightarrow (0, 1)$

$\mu_A^{unknown}(x) = 1$

$\mu_A^{FCF}(x) = 1 - \mu_A^{known}(x)$

$\mu_{fly}^{FCF}(x) = \{1 - \mu_{fly}^{known}(x)\}$

= {1.0/penguin +1.0/Ozzie+ 1.0/parrot+ 1.0/waterfowl + 1.0/eagle - 0.9/penguin +0.7/Ozzie+ . 0.3/parrot+ 0.15/waterfowl + 0.1/eagle }

= { 0.0/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }

For instance " x can fly" for $\alpha >= 0.5$

Is given as

{ 0.0/penguin+0.0/Ozzie+1/parrot+0.65 /waterfowl + 1/eagle }

The inference is given by

Penguin and Ozzie can't fly

Parrot, waterfowl and eagle can fly

## V. FUZZY NON-MONOTONIC LOGIC

Formation of the fuzzy non-monotonic logic is simply two fold fuzzy logic , the non-monotonic proposition may be represented with two fold fuzzy set

$\mu_P(x) = \{\mu_P^{unknown}(x), \mu_P^{nunknown}(x)\}$

For instance,

$\mu_{bird}(x) = \{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\}$

where $\mu_P^{unknown}(x)=1$

$\mu_{bird}^{FCF}(x) = \{1 - \mu_{bird}^{known}(x)\}$

= {1.0/penguin +1.0/Ozzie+ 1.0/parrot+ 1.0/waterfowl + 1.0/eagle - 0.9/penguin +0.7/Ozzie+ . 0.3/parrot+ 0.15/waterfowl + 0.1/eagle }

= { 0.0/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }

$\mu_{bird}(x) = \{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\}$

$\mu_{bird}^{FCF}(x) = \{1 - \mu_{bird}^{known}(x)\}$

if x is $P_1$ and x is $P_2$ …. x is $P_n$ then x is Q

if some information is added to the propostion than inference will be changed.

if x is $P_1$ and x is $P_2$ …. x is $P_n$ anx x is $P_{n+1}$ then x is $Q_1$

x is bird Λ x has wings Λ x is known to fly → x can fly

x is bird Λ x has wings Λ x is unknown to fly → x can or can't fly

The two statements combined with two fold fuzzy logic.

$\mu_{bird}(x) \wedge \mu_{wings}(x) \rightarrow \mu_{fly}(x)$

$\{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\} \wedge \{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\} = \mu_{fly}(x)$

$\mu_{bird}^{FCF}(x) \wedge \mu_{wings}^{FCF}(x) \rightarrow \mu_{fly}(x)$

if x is $P_1$ and $P_2$ …x is $P_n$ then x is $P_1$ and $P_2$ …. x is $P_n$ = $\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$

$\mu_{fly}(x) = \mu_{bird}^{FCF}(x) \wedge \mu_{wings}^{FCF}(x)$

$\mu_{bird}^{FCF}(x)$ = {1.0/penguin +1.0/Ozzie+ . 0.8/parrot+ 0.85/waterfowl + 0.9/eagle – 0.9/penguin +0.8/Ozzie+ . 0.2/parrot+ 0.1/waterfowl + 0.05/eagle }

= {0.1/penguin +0.2/Ozzie+ . 0.6/parrot+ 0.75/waterfowl + 0.8/eagle }

$\mu_{wings}(x) = \{\mu_{wings}^{unknown}(x), \mu_{wings}^{unnown}(x)\}$

$\mu_{wings}(x) = 1$, where wngs is quasi fuzzy set

Reddy fuzzy conditional inference "consequent part "may be derived from "precedent part".

The consequent part is derived from precedent part.

Using (2.3), the fuzzy conditional inference is given by

$\mu_P(x) \wedge \mu_Q(x) \rightarrow \mu_S(x)$

$\mu_S(x) = \mu_P(x) \wedge \mu_Q(x)$

x is bird Λ x has wings → x willfly

x will fly =min { x is bird , x has wings }

The inference of " x can fly" for $\alpha >= 0.5$ is given by

= 1/parrot+ 1/waterfowl + 1/eagle

The parrot, waterfowl and eagle can fly.

The penguin and Ozzie can't fly

Here fuzzy logic made imprecise information to precise information's. Some birds can fly and some birds can't fly.

The fuzzy decision sets or quasi fyzzy set is defined by

$R = \mu_A^R(x) = 1 \quad \mu_A^{FCF}(x) \leq \alpha,$

$\qquad 0 \quad \mu_A^{FCF}(x) > \alpha$

The inference of " x can.t fly" for $\alpha < 0.5$ is given by

= 0/.1penguin +0.2/Ozzie

The inference of " x can fly" for $\alpha >= 0.5$ is given by

= 0.6/parrot+ 0.75/waterfowl + 0.8/eagle

he parrot, waterfowl and eagle can fly and, penguin and Ozzie are can't fly.

## VI. FUZZY TRUTH MAITANACE SYSTEM

Doyel [4] studied truth maintenance system TMS] for non-monotonic reasoning

The fuzzy truth maintenance system (FTMS) for fuzzy non-monotonic reasoning using fuzzy conditional inference as

if x is x is $P_1$ and $P_2$ …. And x is $P_n$ then Q

$= \min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$

FTMS is having There is list of justification and conditions.

if x is bird and x has wings then x can fly

List L(IN-node, OUT-node)

IN node is unknown information.

OUT node is known information.

L1 bird(unknown,known)
L2 wings(unknown-, known)

The concequent part derived from precedent patrt i.e, fly

The FTMS gives using FCF
L1 bird(1, 0.3)
L2 wongs(1, 0.4)
Fly= min{0.7, 0.6}=0.6
fly if FCF>0.5
The FTMS gives using FCF
L1 bird(1, 0.5)
L2 wongs(1, 0.6)
Fly= min{05,0.4}=0.6
can' fly if FCF<0.5

in the case of parrot, waterfowl and agle can fly
in case of Penguin and Ozzie can't fly.

Here the fuzzy nonmonotonic logic is making uncertainty to certainty.

Consiser another example,
"Raju and Rani are in the house. There is gun. Rani died with gun shot." Whether Raju shot at Rani or Rani shot herself.

if x is shoot her and x is shoot by herself and investigation then x is killed

The FTMS gives using FCF
L1 shoot-by-herself(1, 0.7)
L2 shot her(1, 0.6)
L2 investigation(1, 0.8)
killed= min{3, 0.4,0.2}=02
Not killed if FCF<0.5
The FTMS gives using FCF
L1 shoot-by-herself(1, 0.3)
L2 shot her(1, 0.4)
L2 investigation(1, 0.2)
killed= min{0.7,0.6,0.8}=0.6
killed if FCF>=5

## VII. FUZZY MODULATIONS AND LOGIC PROGRAMMING

The fuzzy reasoning system(FRS) is complex reasoning system for incomplete AI problem solving. The fuzzy predicate logic (FPL) is modulating transform fuzzy facts and rules in to meta form(semantic form). These fuzzy facts and rules are modulated to represent the knowledge available to the incomplete problem [17].

The fuzzy modulations for Knowledge representation are type of modules for fuzzy propositions "x is A".

"x is A" is may be represented as
[A]R(x),
where A is twofold fuzzy set {unknown, known}, R is relation and x is individual in the Unversed of discourse X.
For instance
"x is bird" is modulated as
[bird]is(x)

The FPL is e combined with logical operators.
Let A and B be two fold fuzzy sets.
x is ¬A
[¬A]R(x)
x is A or x is B
[A V B]R(x)
x is A and x is B
[A Λ B]R(x)
if x is A then x is B
[A → B]R(x)

x is bird
[bird]is(x)
if x is bird then x can fly
if [bird]is(x) then [fly]is(x)
or
[bird]→[fly]is(x)
if x is bird and x has wings then x can fly
if [bird]is(x)Λ[wings]has[x] then [fly]can(x)

if x is x is $P_1$ and $P_2$ …. And x is $P_n$ then Q = { $\mu_Q(x)$}
if [bird]is(x)Λ[wings]has[x] then [fly]can(x)
[fly]can(x)= { [bird]is(x)Λ[wings]has[x]

The Logic Programming may be written in SWI-Prolog as

```
fuzzy(Ozzie, A,B, M) :- A<B, M is A.
fuzzy(Ozzie, A,B, M) :- A>=B, M is B.
fuzzy(C,M,F):-C<M,F is C.
fuzzy(C,M,F):-C >=M F is M.
fuzzy(X, A,B,C,F) :- fuzzy(X, A,B,M), fuzzy(C,M,F).
?-run(X,0.8,0.7,0.64,F).
F=0.64
If F >=6, Ozzie can fly
?-run(Ozzie,0.6,0.5,0.4,F).
F=0.4
If F <6, Ozzie can't fly
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