Proof of CollatzTheorem

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Abstract

In this article, we will show that Collatz is theorem and we proof it by method that we made in section 2 and 3. In section 1, first we introduction Collatz problem and idea of mathematician about this problem then we change the function of this problem and we make a new definition of Collatz set and generalize Collatz problem in the set theory. In section 2 we decrease all of natural numbers to $\mathbb{Z}_{10}$ and make a model with lemma that we said. Then in section 3 we say 3 properties of numbers that are in our models and then we make a new definition of coloring of graph to complete our model and make a new model to explain Collatz system with 3 numbers. Finally in section 4 we begin proof some part of first model and we use properties that we proved in section 3, to proof our model completely.

Keywords: Collatz, Number theory, conjecture, Proof of Collatz, Graph theory.

1 Introduction

The Collatz conjecture is a conjecture in mathematics that concerns a sequence defined as follows: start with any positive integer n. Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half of the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of n, the sequence will always reach 1.[1]

The conjecture is named after Lothar Collatz, who introduced the idea in 1937, two years after receiving his doctorate.[2] It is also known as the $3n + 1$ problem, the $3n + 1$ conjecture, the Ulam conjecture (after Stanisław Ulam), Kakutani's problem (after Shizuo Kakutani), the Thwaites conjecture (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), or the Syracuse problem.[3][4]

Paul Erdős said about the Collatz conjecture: "Mathematics may not be ready for such problems."[6] Jeffrey Lagarias stated in 2010 that the Collatz conjecture "is an extraordinarily difficult problem, completely out of reach of present day mathematics."[7]

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So Collatz conjecture is a problem that no one has been able to solve completely for the past 80 years. This problem states that:

If, in a set of natural numbers, we multiply each odd number by three and add it to a sum and divide each even number by two, it finally comes to one after a certain number of steps. In other words:

$$\forall x \in \mathbb{N}^1, f^n(x) = 1 \iff f(x) = \begin{cases} 3x+1 \iff x \in \mathbb{O} \\ \frac{x}{2} \iff x \in \mathbb{E} \end{cases}$$

In other words we can say it as: if $f(x) = \begin{cases} 3x+1 \iff x \in \mathbb{O} \\ \frac{x}{2} \iff x \in \mathbb{E} \end{cases}$ rule be established, will any natural number reach 1 or we have a different value that for it, function can't reach 1?

By trying Collatz on several numbers, we find that: the number one has the infinite period in the Collatz conjecture because: $1 \to 4 \to 2 \to 1 \to 4 \to \cdots$ and this period of rotation never ends.

The problem with this periodicity is that any number that reaches one experiences an infinite periodicity.

To eliminate the periodicity, we extend the properties of Collatz as follows:

$$g(x) = \begin{cases} 3x+1 \iff x \in (\mathbb{O} - \{1\}) \\ \frac{x}{2} \iff x \in \mathbb{E} \\ 1 \iff x = 1 \end{cases}$$

If we want to generalize the definition of Collatz we can say:

Definition 1.2. If a number such as x reach 1 with $g(x)$ function then we say that it's in set of truths or in other words:

$$x : \text{reach 1 with } g(x) \text{ function} \Rightarrow x \in T$$

Or we can define T as: $T = \{x|x \text{ reach 1 with } g(x) \text{ function}\}$

And now set of false is a set numbers which are not in T, are in it, or in other word: $F = T' = \{x|x \notin T\}$

To proof Collatz with this definition, we must show that any natural number is in T set.

In Section 2, we create a graph that reduces the infinite number of natural numbers to 10 by several lemma and theorems. We must prove that no number has an infinite period, which is possible in Section 4. (In Section 3, we reduce 10 to 3 to make proofs more straightforward and easier.)

2 Making graph to describe the Collatz system

In order to draw a graph for the Collatz system, we have to use lemma and draw this graph through them.

If we put an odd number in f, the result will be even:

$$x = 2k+1 \Rightarrow 3x+1 = 6k+4 = 2K$$

But if the number is even, the result can not be said, because:

[1]. $\mathbb{N} = \{1, 2, 3, 4, ...\}$

$$x = 2k \Rightarrow \frac{x}{2} = k$$

Because it's not possible to say k we will give two lemma to say what will happen to k.

Lemma 2.1. Suppose that $x = \sum_{i=1}^{n} a_i{}^2 10^{i-1}$ is given number. As $a_i \in \{x \in \mathbb{W}^3, 0 \leq x \leq 9\}$ and x is odd then $f(x)$ will become even if and just if $f(a_1)$ become even, or in other words:

$$x = \sum_{i=1}^{n} a_i 10^{i-1}, x = 2k+1 \Rightarrow f(x) = 2K \iff f(a_1) = 2A_1$$

Proof. To prove the given lemma, it is sufficient to place the rule in the Collatz system, since x is an odd, then a_1 will be an odd, so:

$$a_1 = 2q+1, 0 \leq q \leq 4 \Rightarrow f(\sum_{i=1}^{n} a_i 10^{i-1}) = 3\sum_{i=1}^{n} a_i 10^{i-1} + 1$$

Now with extensive writing we will have to:

$$3\sum_{i=1}^{n} a_i 10^{i-1} + 1 = 3\sum_{i=2}^{n} a_i 10^{i-1} + 3a_1 + 1 =$$
$$3\sum_{i=2}^{n} a_i 10^{i-1} + 3(2q+1) + 1 = 3\sum_{i=2}^{n} a_i 10^{i-1} + 6q + 4$$

So $6q + 4 = 2A \equiv 2A_1 (\text{mod} 10)$ is new unit that it's even so lemma is true.■[4]

But we can not say about a number that is even, because we can not say that k is even or odd. Following lemma provide something that tell what will happen to k.

Lemma 2.2. Suppose that $x = \sum_{i=1}^{n} a_i 10^{i-1}$ is given number. As $a_i \in \{x \in \mathbb{W} | 0 \leq x \leq 9\}$ then in $f(x)$, new $a_1$become in the form of $\frac{a_1}{2} + 5$ if and just if old a_2 is odd and new $a_1$become in the form of $\frac{a_1}{2}$ if and just if old a_2 is even, or in other words:

$$\frac{x}{2} = \sum_{i=1}^{n} b_i 10^{i-1} \Rightarrow \begin{cases} b_1 = \frac{a_1}{2} \iff a_2 = 2k \\ b_1 = \frac{a_1}{2} + 5 \iff a_2 = 2k+1 \end{cases}$$

Proof. In the first state we choose even a_2:

$$x = \sum_{i=1}^{n} a_i 10^{i-1} = \sum_{i=3}^{n} a_i 10^{i-1} + 10a_2 + a_1 = \sum_{i=3}^{n} a_i 10^{i-1} + 20A_2 + a_1$$

$$\frac{x}{2} = (\sum_{i=3}^{n} a_i 10^{i-1} + 20A_2 + a_1)/2 = (\sum_{i=3}^{n} a_i 10^{i-1})/2 + 10A_2 + \frac{a_1}{2}$$

$\therefore b_1 = \frac{a_1}{2}$

So the first state is true. Now let choose odd a_2:

$$x = \sum_{i=1}^{n} a_i 10^{i-1} = \sum_{i=3}^{n} a_i 10^{i-1} + 10a_2 + a_1 = \sum_{i=3}^{n} a_i 10^{i-1} + 20A_2 + 10 + a_1$$

$$\frac{x}{2} = (\sum_{i=3}^{n} a_i 10^{i-1} + 20A_2 + 10 + a_1)/2 = (\sum_{i=3}^{n} a_i 10^{i-1})/2 + 10A_2 + 5 + \frac{a_1}{2}$$

$\therefore b_1 = \frac{a_1}{2} + 5$

So given lemma is true.■[5]

If we want to make a graph to describe the Collatz theorem, we must note that the field of $\mathbb{Z}_{10}$ is a complete field for describing the Collatz system, because in lemma 2.1 and 2.2 we saw that only by knowing the unit can the future of Collatz be predicted and thus proved.(We do not need to know tens because we know what the next numbers will be during lemma 2.2 .)

Now suppose the graph G is the same as the assumed graph, now we define it as follows:

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[2]. At all of this article a_i is in $0 \leq \forall a_i \leq 9, \forall a_i \in \mathbb{W}$.

[3]. $\mathbb{W} = \{0, 1, 2, 3, \dots\}$

[4]. Finished proof of Lemma 2.1

[5]. Finished proof of Lemma 2.2

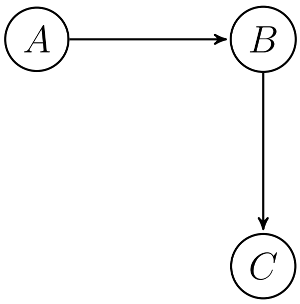
$$G : \begin{cases} V(G) = \{v_i | i \in \mathbb{W}, 0 \leq i \leq 9\}, v_i = i \\ E(G) = \{\overrightarrow{v_i v_j} | i = x, j = y\} \end{cases}$$

The reason for choosing the vertices is that we are looking at one, not the other. (Because it depends on what happens to the unit [we proved this in lemma 2.1 and lemma 2.2 .]).

Now we express the x and y in the edges. Since each edge is defined according to the Collatz instruction, then v_j is defined according to the Collatz rule, which can be accurately defined by lemma 2.1 and 2.2.

Definition 2.1. Degree is the same definition as for a graph, but with a number of differences, the output degree being denoted by $\deg^+$ and the output degree denoted by $\deg^-$.

For example:



In the graph G: $\deg^+(A) = 1$ means that the output degree A is equal to 1.

In the graph G: $\deg^+(B)$ =$\deg^-(B) = 1$, the input degree (def$^-$) and the degree degree ($\deg^+$) are equal.

Now, considering this issue, we state two theorems to create the given graph:

Theorem 2.1. If it is $i \in O$ and we assume v_i as the vertex then:

$$\deg(v_i)^{\pm} = 1$$

Proof. It is clear from one that an odd number will only go to an even number. So according to lemma 2.1, the output degree is completely correct, and now we prove the input degree of the given graph, we know from lemma 2.2 and lamme 2.1 that only even numbers make up an odd number now we show in lemma 2.3 that these numbers are unique and we finish our proof with respect to the input vertex.

Lemma 2.3. For any even x there is only and just only one odd x_1 in $\mathbb{Z}_{10}$.

Proof. Suppose this is not the case, then x and x_0 will reach x_1, so we measure them relative to each other according to lemma 2.2 If we assume $\frac{x}{2} = \frac{x_0}{2}$, the result is correct (because it gives $x = x_0$ when we multiply the sides by 2.)

So suppose the second case is $x/2 = x_0/2 + 5$ and show that the two are also equal:

$$\frac{x}{2} = \frac{x_0}{2} + 5$$

Multiply the sides by 2. And we have:

$$x = x_0 + 10$$

That in a given field:

$$x = x_0$$

So given lemma is true and thus the theorem is proved, because only a single vertex reaches the hypothetical vertex and the hypothetical vertex reaches a unique vertex, the hypothetical is completely wrong.■[6]■[7]

Theorem 2.2. If it is $i \in E$ and we assume v_i as the vertex then:

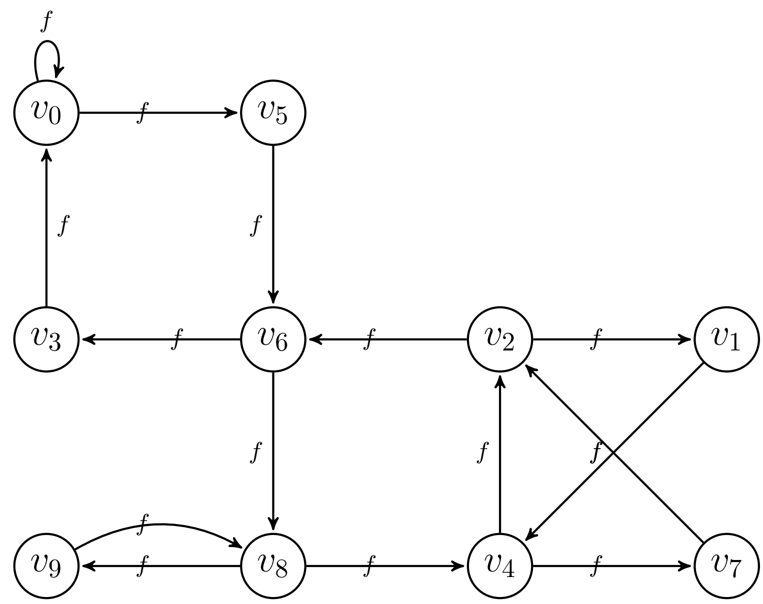
$$\deg(v_i)^{\pm} = 2$$

Proof. It is clear from lemma 2.2 that the output degree is correct. So, considering lemma 2.2, half of the theorem is absolutely correct. Now to prove the degree of input, we will deal with the fact that each odd number will go to an even number, so we only have to prove one thing that any even number will reach an even number, it is unique that when we put an even instead of an odd in lemma 2.3, the problem is completely proven.(This means that according to Lemma 2.2, one of $a_1/2 + 5$ or $a_1/2$ can be even. So this means that any even number can reach another even number, so $\deg^+(v_i)$ $=\deg^-(v_i) = 2, i \in \mathbb{E}$ is correct (because according to Lemma 2.1 a person reaches an even number And according to Lemma 2.2 a pair reaches a pair.))■[8]

Now according to the said theorems any v_j is $f(v_i)$ that they are in $\mathbb{Z}_{10}$ field. For example v_3 with $\overrightarrow{v_3 v_0}$ edges will reach to v_0. So $E(G)$ can be defined as follows:

$$E(G) = \{\overrightarrow{v_i v_j} | f(v_i) \equiv v_j \pmod{10}\}$$

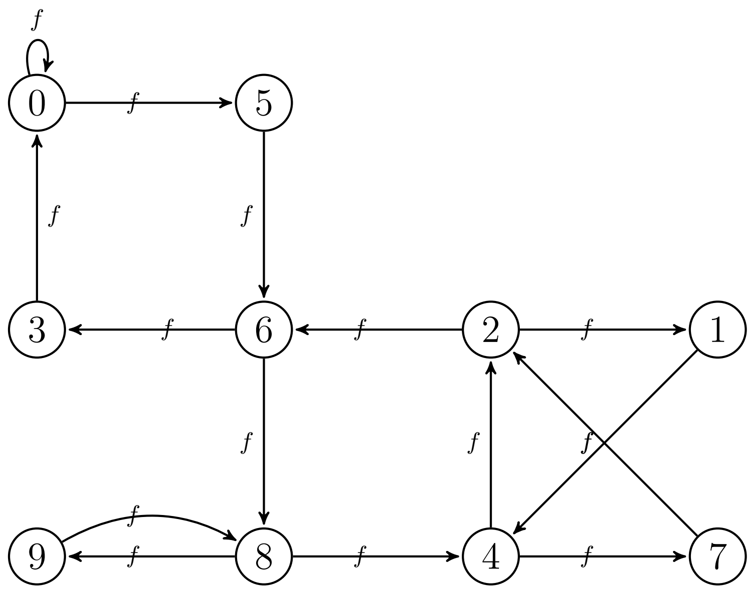
Now, according to the lemmas and theorems, the graph G can be drawn as follows:



Graph 2.1: Graph of Colltaz system in $\mathbb{Z}_{10}$

6. Finished proof of Lemma 2.3
7. Finished proof of Theorem 2.1
8. Finished proof of Theorem 2.2

And as we know that $v_i = i$ so we can have this graph too:



Graph 2.2: Graph of Colltaz system in $\mathbb{Z}_{10}$

3 Coloring and numbers properties

Now here we will reduce 10 numbers (that we reduced them to 10 in section 2) to 3 numbers that have the same properties.

In sub-section 3.1, we will talk about numbers properties and we will prove their propertirs.

Then in sub-section 3.2, we will make a new graph with a new definition of coloring that we made there.

3.1 Numbers properties and sets

Theorem 3.1.1. According to lemma 2.1, we can make a set that it makes of odd numbers. (Because according to Lem 2.1, all odd numbers have the same property):

$$S_1 = \{1, 3, 5, 7, 9\}$$

Proof. According to lemma 2.1, these numbers determine what will happen to the numbers of the natural person of our choice. ($\forall x \in \mathbb{O} \Rightarrow \forall f(x) \in \mathbb{E} \iff \forall a_1 \in \mathbb{O} \Rightarrow \forall f(a_1) \in \mathbb{E}$)
Since each odd number reaches another even number, and this number is necessarily different, the theorem is clear and correct.$\blacksquare$[9]
Theorem 3.1.2. According to lemma 2.2, we can create two sets, each with distinct properties:

$$S_2 = \{0, 4, 8\}$$
$$S_3 = \{2, 6\}$$

[9]. Finished proof of Theorem 3.1.1

Proof. To prove the theorem, we examine and prove the properties one by one in two distinct theorems.

Theorem 3.1.3. S_2 has the following properties:

$$x = \sum_{i=2}^{n} a_i 10^{i-1} + a_1, a_1 \in S_2 \Rightarrow \begin{cases} \text{(i) } f(a_1) \in \mathbb{E} \iff a_2 \in \mathbb{E} \\ \text{(ii) } f(a_1) \in \mathbb{O} \iff a_2 \in \mathbb{O} \end{cases}$$

Proof. (i) According to lemma 2.2: $f(a_1) = a_1/2$

Definition 3.1.1. S_2 can be defined as $S_2 = \{x|x = 4k, 0 \leq k \leq 2, \forall k \in \mathbb{W}\}$. Since $a_1 \in S_2$ then it can be concluded that:

$$a_1 \in S_2 \Rightarrow a_1 = 4k$$
$$f(a_1) = f(4k) = \tfrac{4k}{2} = 2k \Rightarrow a_2, 2k \in \mathbb{E}$$

According to the final result, the first part of the proposition was proved.

(ii) According to lemma 2.2: $f(a_1) = \frac{a_1}{2} + 5$

According to definition 3.1.1, we will have:

$$a_1 \in S_2 \Rightarrow a_1 = 4k$$
$$f(a_1) = f(4k) = \tfrac{4k}{2} + 5 = 2k + 5 = 2(k+2) + 1 = 2k' + 1 \Rightarrow a_2, 2k' + 1 \in \mathbb{O}$$

According to the final result, the proposition was proved and by proving it, the theorem was also proved.■[10]

Theorem 3.1.4. S_3 has the following properties:

$$x = \sum_{i=2}^{n} a_i 10^{i-1} + a_1, a_1 \in S_3 \Rightarrow \begin{cases} \text{(i) } f(a_1) \in \mathbb{O} \iff a_2 \in \mathbb{E} \\ \text{(ii) } f(a_1) \in \mathbb{E} \iff a_2 \in \mathbb{O} \end{cases}$$

Proof. (i) According to lemma 2.2: $f(a_1) = a_1/2$

Definition 3.1.2. S_3 can be defined as $S_3 = \{x|x = 4k+2, 0 \leq k \leq 1, \forall k \in \mathbb{W}\}$. Since $a_1 \in S_3$ then it can be concluded that:

$$a_1 \in S_3 \Rightarrow a_1 = 4k + 2$$
$$f(a_1) = f(4k+2) = \tfrac{4k+2}{2} = 2k + 1$$
$$\therefore a_2 \in \mathbb{E}, f(a_1) = 2k + 1 \in \mathbb{O}$$

(ii) According to lemma 2.2: $f(a_1) = \frac{a_1}{2} + 5$

According to definition 3.1.2, we will have:

$$a_1 \in S_3 \Rightarrow a_1 = 4k + 2$$
$$f(a_1) = f(4k+2) = \tfrac{4k+2}{2} + 5 = 2k + 6 = 2(k+3) = 2k'$$
$$\therefore a_2 \in \mathbb{O}, f(a_1) = 2k' \in \mathbb{E}$$

According to the results, Theorem 3.1.4 and Theorem 3.1.2 are proved and are absolutely correct.■[11]■[12]

[10]. Finished proof of Theorem 3.1.3

[11]. Finished proof of Theorem 3.1.4

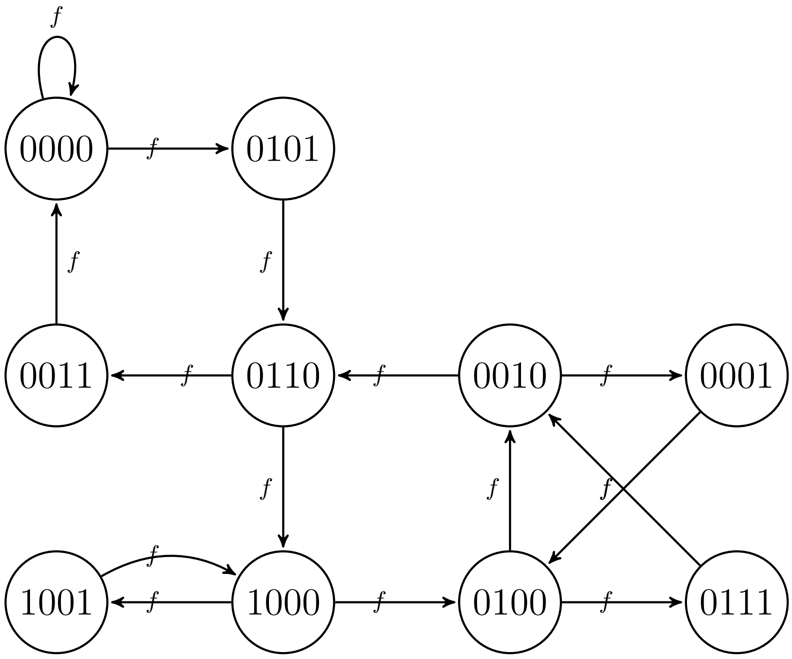
[12]. Finished proof of Theorem 3.1.2

3.2 Binary arithmetic (binary system) and coloring

It is very interesting that the number of sets (S_1, S_2, S_3) is exactly equal to the maximum number of coloring G $(\chi(G))$.

We can only coloring G with three colors. For this reason, we can not calculate and consider all coloring schemes.

Now we place each graph number in its binary state. We will see that G_{Binary} is:

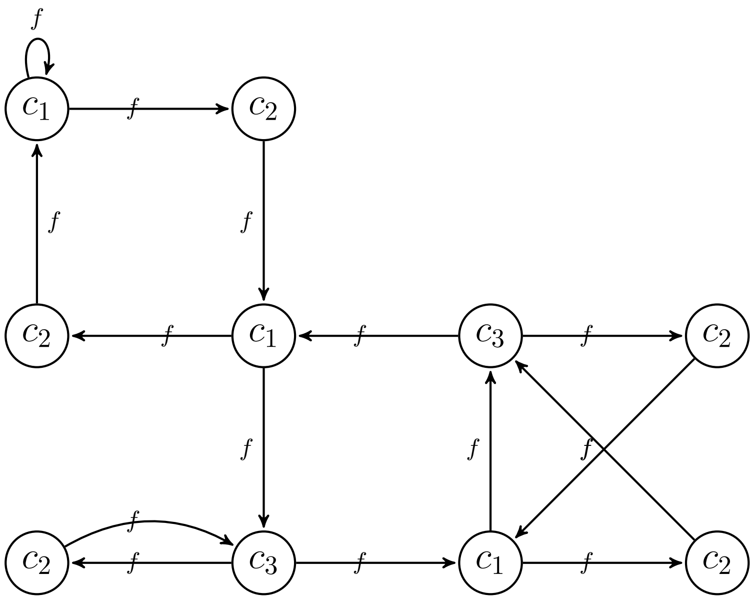


Graph 3.2.1: Graph of Colltaz system in binary arithmetic.

We create an easy rule with a binary arithmetic for coloring graphs and that rule is:

$$c_i, i \in \mathbb{E} \Rightarrow j \in G_{Binary}, j \in \mathbb{O}$$
$$c_i, i \in \mathbb{O} \Rightarrow j \in G_{Binary}, j \in \mathbb{E}$$

According to the law, a graph can be colored similar to the graph below:



Graph 3.2.2: Coloring system of Collatz graph

Of course, this is one of the forms of coloring according to those two rules.

With a little attention to the graph and its comparison with the theorems that proved the properties of even numbers, we will have:

$$\text{Due to Graph 3.2.2} \Rightarrow \text{(i) } v_6 = c_1, v_2 = c_3$$
$$\text{Due to subsection 3.1} \Rightarrow \text{(ii) } v_6 \text{ and } v_2 \text{ have common properties}$$
$$\text{Due to our rule} \Rightarrow \text{(iii) } c_1 \text{ and } c_2 \text{ heve not common properties}$$
$$(\text{(i)} \not\Leftrightarrow \text{(ii), (iii)}) \Rightarrow \therefore \text{ The assumed graph coloring is incorrect}$$

So we have to coloring the graph in a different way and according to the rule we said, but since $\overrightarrow{v_2 v_6}$ are next to each other, the type of coloring will always be different, and this creates a paradox that can not be used to continue coloring.

Because of these events, violations, and problems, we need to create a new definition of coloring.

We know that our definition of coloring is as follows:

Definition 3.2.1. (Normal type coloring (C)). We say that C is the coloring of the graph G if and just if the two vertices that are connected to each other in G by an edges have different colors.

The above definition is the simplest definition of graph coloring.

Now we create a new type of coloring by breaking some of the rules of coloring.

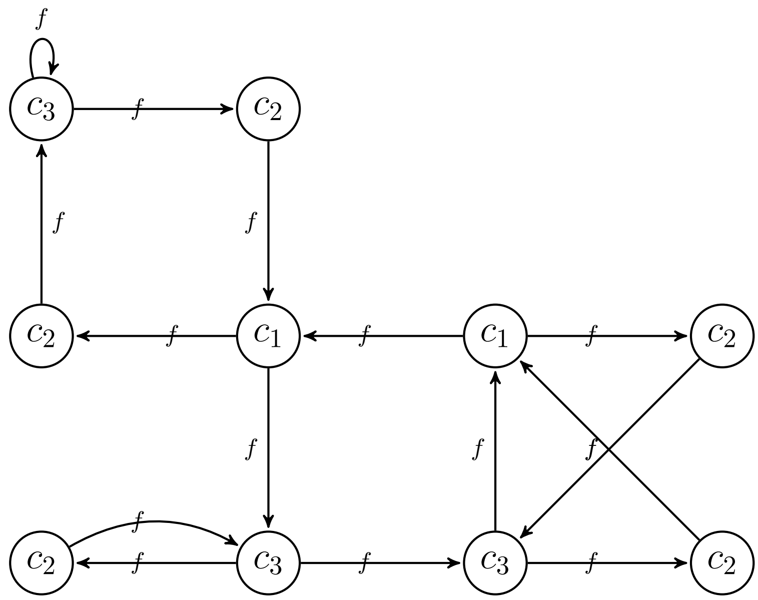
Definition 3.2.2. (Collatz type coloring ($C_{Collatz}$)). We say that the graph of G is coloring as $C_{Collatz}$ if and only if $C_{Collatz}$ is coloring based on the properties of G and must be coloring with the minimum ($\chi(G)$) allowable amount of G coloring, and each member G define a directional graph, and in its coloring, two vertices connected by an edge do not necessarily have to have distinct colors.

By generalizing definition 2, we will have:

Definition 3.2.3. (Beta type coloring (C_β). If $C_{Collatz}$ is for graphs that have no direction, C_β is defined.

One of the properties that we find according to the definition of $C_{Collatz}$ and C_β is that: it is not necessary to coloring the two vertices that are next to each other, and this coloring should only be based on the properties and sets and no more than $\chi(G)$

If we want to coloring the graph based on $C_{Collatz}$ or C_β, we will have:



Graph 3.2.3: $C_{Collatz}$ (C_β) Coloring system of Collatz graph

And the result of this graph is as follows:

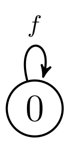
$$\forall c_2 \in S_1, \forall c_3 \in S_2, \forall c_1 \in S_3$$

4 Proof of Collatz theorem

In this section, we first prove part of the Collatz conjecture by the graph we created in section 2, and then we prove the Collatz by the graph we created in section 3 and the question variations we created in section 1 in the definition of Collatz conjecture.

4.1 Proof of Collatz conjecture with graph 2.1

Theorem 4.1.1. If G_0 is defined as $G_0 : \begin{cases} V(G_0) = \{v_0\} \\ E(G_0) = E(G[V(G_0)]) \end{cases}$ then G_0 is true in Collatz (it is in the set of truths), or in other words:

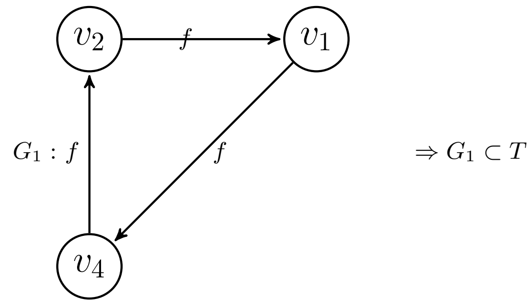
G_0 :  $\Rightarrow G_0 \subset T$

Proof. For proof given theorem assume the number n is in v_0. During the definitions of the assumed graph in 2, n is defined in the form of $n = \sum_{i=2}^{k} a_i 10^{i-1} + 10$. Or the definition of n can be generalized to $n = 2^a \cdot 5^b \cdot (2k+1)$. (The third number that is multiplied must be odd because if it is not odd then it can be multiplied by increasing the power of 2.) In Collatz function, n, after a steps, finally will reach to: $5^b(2k+1)$. Now we put new number in our field ($\mathbb{Z}_{10}$ field) and:

$$5^b(2k+1) \equiv 10 \cdot 5^{b-1}k + 5^b \equiv 5^b \equiv 5 \pmod{10}$$

So it reach to 5 and theorem proof. So $G_0 \subset T$.■[13]

Theorem 4.1.2. If G_1 is defined as $G_1 : \begin{cases} V(G_1) = \{v_1, v_2, v_4\} \\ E(G_1) = E(G[V(G_1)]) \end{cases}$ then G_1 is true in Collatz (it is in the set of truths), or in other words:



Proof. First assume that G_1 is in F for a distinct value.($\exists x \in G_1 \Rightarrow x \in F$) Now we will show that this distinct value will not be except 1 and negative numbers and we will err in the proposition and thus we will conclude that the theorem is true.
We begin with a number that it is in $x = \sigma + 1 = \sum_{i=2}^{n} a_i 10^{i-1} + 1$ form. After 3 steps with calculating, it will become in $\frac{3\sigma}{4} + 1$ form:

$$f(\sum_{i=2}^{n} a_i 10^{i-1} + 1) \to f(3\sum_{i=2}^{n} a_i 10^{i-1} + 4) \to f(\frac{3\sum_{i=2}^{n} a_i 10^{i-1}}{2} + 2) \to \frac{3\sum_{i=2}^{n} a_i 10^{i-1}}{4} + 1 = \frac{3\sigma}{4} + 1$$

We know that $\forall x \in \mathbb{O} : \frac{3x+1}{4} \leq x$, $\frac{3x+1}{4} = x \Rightarrow x = 1$:

$$\frac{3x+1}{4} = x$$
$$\Rightarrow 3x + 1 = 4x$$
$$\Rightarrow 1 = 4x - 3x = x$$

And if $\frac{3x+1}{4} > x \Rightarrow x < 1$:

$$\frac{3x+1}{4} > x$$
$$\Rightarrow 3x + 1 > 4x$$
$$\Rightarrow 1 > 4x - 3x = x$$

And as we told before, 1 x (because it makes an infinity ring). And negative integers can't too.(because our numbers is in $\mathbb{N}$.) So equal and bigger part it not true:

[13]. Finished proof of Theorem 4.1.1

$$\frac{3x}{4} + \frac{1}{4} < x, \forall x \in \mathbb{O} \setminus \{1\}$$

This inequality shows that: numbers will become lesser than, untill it becomes equal 1 (because we can say if it is false so it becomes infinity ring) and due to g(x) function, we will see that 1 will reach 1 so this graph is true for any $\mathbb{N}$ number or we can say: $x \in \mathbb{N}, x \in G_1 \Rightarrow x \in T$. So $G_1 \subset T$.■[14]

4.2 Prove Collatz's conjecture by coloring the graph (graph 3.2.3)

In theorem 4.1.2 and theorem 4.1.1, we proved that $G_0, G_1 \subset T$ and due to logic of propositions, we will have:

$$\forall x \in G_0, G_1 \Rightarrow x \in T \text{ or: } \overrightarrow{c_3c_3}, \overrightarrow{c_2c_3}, \overrightarrow{c_1c_2}, \overrightarrow{c_2c_1}, c_1, c_2, c_3 \in T$$

Because G_0, G_1 is based of $E(G_0), E(G_1), V(G_0), V(G_1)$, we can say all of their (E, V) members are in T. $(G_0 = E(G_0) \cup V(G_0), G_1 = E(G_1) \cup V(G_1) \therefore G_0, G_1 \subset T \iff E(G_0), E(G_1), V(G_0), V(G_1) \subset T)$
Now according this, we can say any sub-graph is true because edges of sub-graphs must have c_1 or c_2 or c_3 and because of that, we proved all of them.
For making sure, we will make a theorem and then we will proof it.
Theorem 4.2.1. Any given sub-graph of graph G is true in Collatz.
Proof. Hypothesis that theorem is not true and we have an unique sub-graph that is not in truths set and we call it G_k.
G_k is defined as $G_k : \begin{cases} V(G_k) = \{v_i | i \text{have some properties}\} \\ E(G_k = E(G[V(G_k)]) \end{cases}$

And if G_k is in F so we will see that:

$$E(G_k), V(G_k) \subset F$$

And due to we proved that any c_1, c_2, c_3 is in T, second part $(V(G_k) \subset F)$ is wrong and if we see graph we will in any sub-graph edges, we have $\overrightarrow{v_3v_3}$ or $\overrightarrow{v_1v_2}$ or $\overrightarrow{v_2v_1}$or $\overrightarrow{v_2v_3}$ and we proved that these edges are in T, so first part $(V(G_k))$ is wrong too.

So we can understand that this hypothesis is not true, so we will see that:

$$E(G_k), V(G_k) \subset F \equiv F \sim G_k \not\subseteq F \sim G_k \subset F' = T$$

So our theorem is true.■[15]

Due to this theorem we will see: $\bigcup_{i=1}^{k} G_i = G \subset T$

Now we propose the last theorem:
Theorem 4.2.2. (Collatz theorem). Any $\mathbb{N}$ number will reach 1 with $g(x) = \begin{cases} 3x + 1 \iff x \in \mathbb{O} \setminus \{1\} \\ \frac{x}{2} \iff x \in \mathbb{E} \\ 1 \iff x = 1 \end{cases}$ function after some steps.
Proof. Hypothesis that the theorem is incorrect and does not reach one for a distinct value of x, and that one enters an infinite periodicity in which one is not.

[14]. Finished proof of Theorem 4.1.2
[15]. Finished proof of Theorem 4.2.1

This x must be part of natural numbers, so its unit must be variable from 0 to 9, and since we have already proved that any natural number whose unit is between 0 and 9 does not form an infinite loop and is true. So x must necessarily be true and not create an infinite loop, thus proving.■[16]

References

[1] https://en.wikipedia.org/wiki/Collatz_conjecture

[2] O'Connor, J.J.; Robertson, E.F. (2006). "Lothar Collatz". St Andrews University School of Mathematics and Statistics, Scotland.

[3] Maddux, Cleborne D.; Johnson, D. Lamont (1997). Logo: A Retrospective. New York: Haworth Press. p. 160. ISBN 0-7890-0374-0. The problem is also known by several other names, including: "Ulam's conjecture, the Hailstone problem, the Syracuse problem, Kakutani's problem, Hasse's algorithm, and the Collatz problem."

[4] According to Lagarias (1985),[5] p. 4, the name "Syracuse problem" was proposed by Hasse in the 1950s, during a visit to Syracuse University.

[5] Lagarias, Jeffrey C. (1985). "The 3x + 1 problem and its generalizations". The American Mathematical Monthly. 92 (1): 3–23. doi:10.1080/00029890.1985.11971528. JSTOR 2322189.

[6] Guy, Richard K. (2004). ""E17: Permutation Sequences"". Unsolved problems in number theory (3rd ed.). Springer-Verlag. pp. 336–7. ISBN 0-387-20860-7. Zbl 1058.11001.

[7] Lagarias, Jeffrey C., ed. (2010). The ultimate challenge: the 3x + 1 problem. Providence, R.I.: American Mathematical Society. p. 4. ISBN 978-0821849408.

[16]. Finished proof of Theorem 4.2.2