A Discrete Time Retrial Queueing System Starting Failures, Bernoulli Feedback with General Retrial Times and a Vacation

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A DISCRETE TIME RETRIAL QUEUEING SYSTEM STARTING FAILURES, BERNOULLI FEEDBACK WITH GENERAL RETRIAL TIMES AND A VACATION

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Abstract

This article is concerned with a discrete time Geo/G/1 retrial queue with general retrial times, Bernoulli feedback and the server subject to starting failures and a vacation. In this article we generalize the previous works in discrete time retrial queue with unreliable serverm due to starting failures in the sense that we consider general service with Bernoulli feed back and general rtrial times with single vacation. In this model arrival time follows geometrical distribution and vacation times are generally distributed. In this model the PGF is derived by using generaing function technique and also we obtain the analytical expression for mean queue length in performance measure. In numerical examples we analyzed the effectis of mean queue length in several possible ways.

Key words

Discrete time retrial queue, Bernoulli feedback, unreliable server, mean queue length, markov chain, single vacation.

1.INTRODUCTION

In Queuing models many researchers have found a lot of application in computer communications and manufacturing systems. Currently many researchers are interested in discrete queue, due to applications in a various slotted digital communicated systems and other related areas. The analysis of discrete queuing model has received considerable attention in the scientific literature over the past years because of its applications which are widely used in the real life.

In telecommunication and computer systems the role of retrial queues in very important and are characterized by the fact that a customer who leaves the service area and joins a retrial group (orbit) when the server is busy. In the customers in the orbit can not receive service immediately, when the server is idle but, it is not so standard queues this is the main difference between retrial and standard queues. Falm (1990), Falm and Templation (1997), Kulkarni and Liang (1997), yangand Templation (1987) have discussed on retrial queues and analyzed the fundamental methods on retrial queues .in brief,

In most of literature the researchers analyzes continuous queuing model, but only some of the authors concentrate on discrete queues since in practice it is applied many systems which shows an inherent genetic slotted time scale (time shared computing system). Initially the discrete queues are discussed by Meisling(1958), Bindsall, Ristenbatt, and Weinstein (1962) and also by powell and Avi – Lizha (1967).. In modelling computers and telecommunications the role of discrete queuing models are most important when compared with continuous time models. The concept feedback is initiated by Takacs (1963) which have been widely investigated in continuous time [5-8,14-16,20] whereas they have been rarely analyzes in discrete time[1].Takacs think about that the number of services needed by a customer is geometrically distributed, that is, after receiving each service a customers quits the system with probability 1-α or rejoins the end of the queue for another service with probability α. This phenomenon o feed back has many practical applications. Also, Atencia, Fortes, and Sanchez (2009) has analyzed a discrete queue with Bernouli feedback and starting failures.

In this article we have developed a new concept in discrete retrial queue deals with Bernoulli feedback, starting failures and a vacation. Since the role of vacation in discrete queue with feedback and starting failures has wide application in many real situations of our life, which is motivated me to develop this article.

The aim of this article is to discuss the problem like that arises in telecommunication systems where messages that produce errors at the destination are sent again in a call centre, where customer may call again (repeat their service) if their problems are completely solved after the service. Also, in the telecommunication system the starting failures occurred and at the time of starting failures no service is produced to customers and the server also takes a vacation of random time. This process is most suitable for our model under consideration. .

2 MODEL DESCRIPITION

This model is concerned with a discrete time Geo/G/1 retrial queue with general retrial times, Bernoulli feedback and the server subject to starting failures and a vacation. In this model a previous work is generalized in discrete time retrial queue with unreliable server due to starting failures in the sense that general service with Bernoulli feedback and general rtrial times with single vacation are considered. Herearrival time follows geometric distribution and service time and vacation time are generally distributed. The PGF is derived by using generaing function technique and also it is obtained an analytical expression for expected queue length in performance measure. In numerical illustrations the effectis of expected queue length is analyzed in several possible ways.

In this model a discrete time queueing system is considered, where the time axis is splitted into constant length interval of one unit called slots. In continuous queue the probability of an arrival and a departure occurring simultaneously is zero, whereas it is not so in discrete queues. Also, in this discrete retrial queue all the queueing activities occur around the slot boundary. The time axis be marked by 0,1,2,…,m consider the epoch m and assume that departures and the end of repair times takes place just before the slot boundary m i.e.,in the interval (m^-, m) and arrivals, retrials, beginning of repairs and vacation takes place just after the slot boundary i.e., in the interval (m,m^+)i.e., an early arrival system or departure first policy is followed.

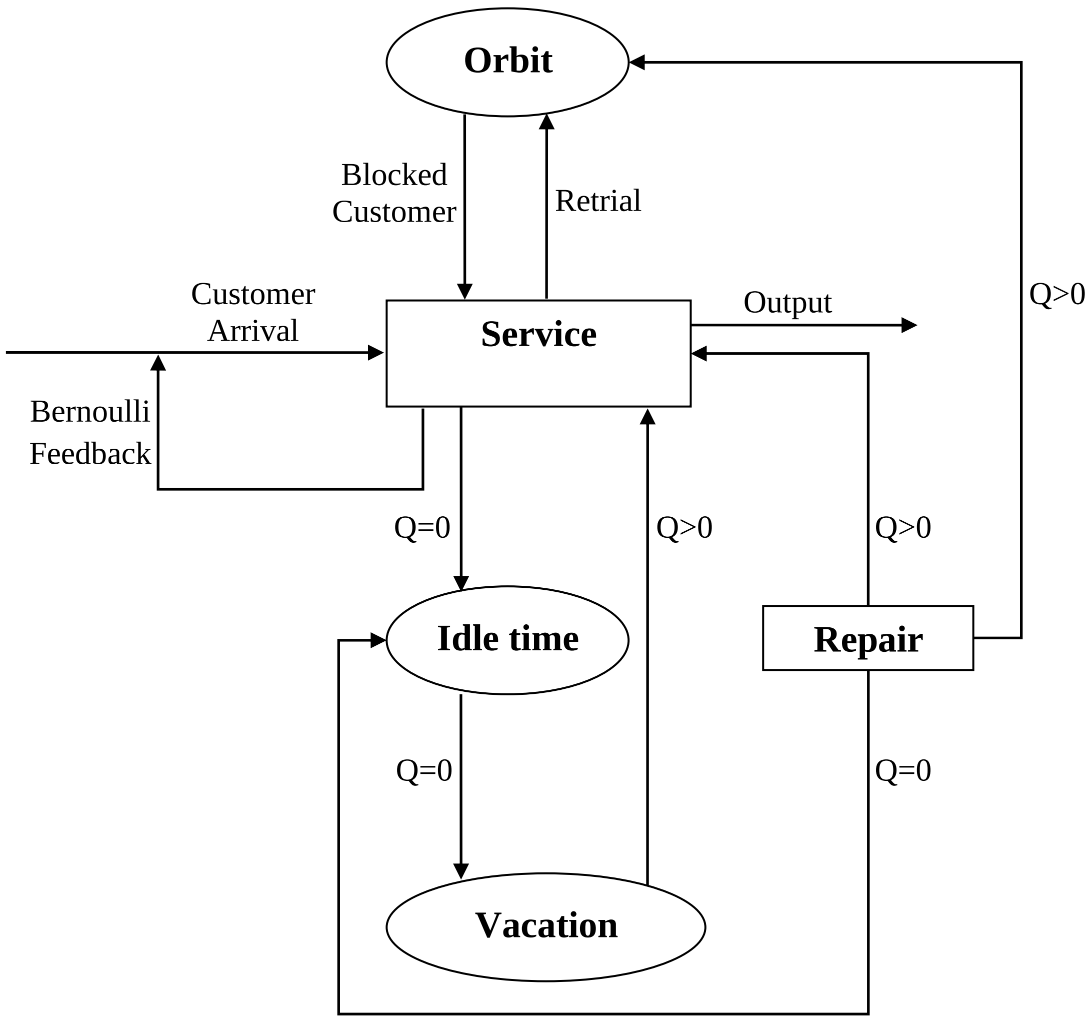

Figure 6.1 Schematic representation of the queueing model

Q: Queue length

The assumptions are as follows,

(1) Customers arrival times follows geometric distribution with probability λ.

(2) The server starts its service successfully with probability p and the server starts its service unsuccessfully with probability $\bar{p} = 1 - p$.

(3) Inter retrial times $\{a_i\}$ are generally distributed and identically independent random variable with cdf A(x) and probability θ.

(4) Service times $\{s_i\}$ are generally distributed and identically independent random variable with cdf S(x).

(5) Repair times $\{r_i\}$ are generally distributed and identically independent random variable with cdf R(x).

(6) Vacation times $\{v_i\}$ are generally distributed and identically independent random variable with cdf $V(x)$.

2.1 NOTATIONS

In this chapter the following notations are used

λ	: Arrival rate
$S(\cdot)$	: Cdf of service time
$V(\cdot)$	: Cdf of vacation time
$A(\cdot)$	: Cdf of retrial time
$R(\cdot)$	: Cdf of repair time
$s(x)$	: Pdf of service time
$v(x)$	: Pdf of vacation time
$a(x)$	: Pdf of retrial time
$r(x)$	: Pdf of repair time
P_m	: State of the system at time m^+
L_m	: Number of repeated customers in retrial at time m^+
$\zeta_{0,m}$	: Remaining vacation time
$\zeta_{1,m}$	: Remaining retrial time
$\zeta_{2,m}$	: Remaining service time
$\zeta_{3,m}$	: Remaining repair time
$P(z)$	: PGF of system size at an arbitrary epoch
$\pi_{0,i,k}$	: Limiting probability of vacation period
$\pi_{1,i,k}$	: Limiting probability of idle period
$\pi_{2,i,k}$	: Limiting probability of busy period
$\pi_{3,i,k}$	: Limiting probability of busy period
g_k	$= P(X=k)$.

State of the system is defined as follows,

P_m = 0, the server is on vacation

= 1, the server is on idle

= 2, the server is on busy

= 3, the server is under repair

3 SYSTEM EQUATIONS

The Kolmogorov equations for the stationary distribution is

$$\pi_{0,1,0} = \lambda\pi_{0,1,0} + \bar{\lambda}\pi_{0,0}, \qquad i \geq 1 \qquad (6.1)$$

$$\pi_{0,i,k} = \bar{\lambda}\pi_{0,i+1,k} + \lambda v_i \pi_{0,1,k} + \bar{\lambda} v_i \pi_{0,1,k+1}, \qquad i \geq 1 \qquad (6.2)$$

$$\pi_{1,1,0} = \bar{\lambda}\pi_{0,0} + \bar{\theta}\bar{\lambda}\pi_{1,1,0}, \qquad i \geq 1 \qquad (6.3)$$

$$\pi_{1,i,k} = \theta\bar{\lambda}a_i\pi_{0,1,k-1} + \bar{\theta}\bar{\lambda}a_i\pi_{0,1,k} + \bar{\lambda}\pi_{1,i+1,k} + \theta\bar{\lambda}a_i\pi_{2,1,k+1} + \bar{\theta}\bar{\lambda}a_i\pi_{2,1,k} + \bar{\lambda}a_i\pi_{3,1,k},$$

$$i \geq 1,\ k \geq 1 \qquad (6.4)$$

$$\pi_{2,i,k} = \lambda s_i\delta_{0k}p\pi_{00} + \lambda p s_i\pi_{0,1,k-1} + \bar{\lambda}ps_i\pi_{0,1,k} + \bar{\lambda}ps_i\pi_{1,1,k+1} + (1-\delta_{0k})p\lambda s_i\sum_{j=1}^{\infty}\pi_{1,j,k} + (1-\delta_{0k})\theta p\lambda s_i\pi_{2,1,k-1}$$
$$+ \left(\bar{\theta}p\lambda s_i + \theta\bar{\lambda}pa_0 s_i\right)\pi_{2,1,k} + \bar{\theta}\bar{\lambda}a_0 ps_i\pi_{2,1,k-1} + (1-\delta_{0k})\pi_{2,i+1,k-1} + \bar{\lambda}\pi_{2,ii+1,k}$$
$$+ (1-\delta_{0k})p\lambda s_i\pi_{3,1,k} + \bar{\lambda}ps_i a_0\pi_{3,1,k+1}, \qquad i \geq 1,\ \ k \geq 0 \qquad (6.5)$$

$$\pi_{3,i,k} = \lambda r_i\bar{p}\delta_{ok}\pi_{00} + \lambda r_i\bar{p}\pi_{0,1,k-1} + \bar{\lambda}r_i\bar{p}\pi_{0,1,k} + (1-\delta_{1k})\lambda\bar{p}r_i\sum_{j=1}^{\infty}\pi_{1,j,k-1} + \bar{p}\bar{\lambda}r_i\pi_{1,1,k} + (1-\delta_{0k})\vartheta\lambda r_i\bar{p}\pi_{2,1,k-2}$$
$$+ \left(\bar{\theta}\bar{p}\lambda r_i + \vartheta\bar{\lambda}a_0 pr_i\right)\pi_{2,1,k-1} + \bar{\theta}\bar{p}\bar{\lambda}a_0 r_i\pi_{2,1,k} + (1-\delta_{1k})\lambda\bar{p}r_i\pi_{3,1,k-1} + \bar{p}\bar{\lambda}a_0\pi_{3,i,k}$$

$$i \geq 1,\ k \geq 1 \qquad (6.6)$$

To solveabove Kolmogorov equations, define generating function's and auxiliary generating function's as follows

$$\phi_0(x,z) = \sum_{i=1}^{\infty}\ \sum_{k=0}^{\infty}\pi_{0,i,k}x^i z^k\ ;\ \phi_1(x,z) = \sum_{i=1}^{\infty}\ \sum_{k=1}^{\infty}\pi_{1,i,k}x^i z^k$$

$$\phi_2(x,z) = \sum_{i=1}^{\infty}\ \sum_{k=0}^{\infty}\pi_{2,i,k}x^i z^k\ ;\ \phi_3(x,z) = \sum_{i=1}^{\infty}\ \sum_{k=1}^{\infty}\pi_{3,i,k}x^i z^k$$

$$\phi_{0,i}(z) = \sum_{k=0}^{\infty}\ \pi_{0,i,k}z^k\ ;\ \phi_{1,i}(z) = \sum_{k=1}^{\infty}\ \pi_{1,i,k}z^k$$

$$\phi_{2,i}(z) = \sum_{k=0}^{\infty}\ \pi_{2,i,k}z^k\ \ and\ \ \phi_{3,i}(z) = \sum_{k=1}^{\infty}\ \pi_{3,i,k}z^k$$

4 STEADY STATE ANALYSIS

In this section the probability generating function of a system size at anarbitrary epoch is obtained by using generating function technique and by using this PGF ananalytical expression for expected queue length is derived.

The limiting probabilities are defined as follows

$$\pi_{0,i,k} = im_{n\to\infty} P\{P_m = 0, \xi_{0,m} = i, L_m = k\}$$

which means that the server is under vacation at time m^+, remaining vacation time $\xi_{0,m}$ is i and number of repeated customers L_m in the orbit is k.

Similarly, define

$$\pi_{1,i,k} = im_{n\to\infty} P\{P_m = 1, \xi_{1,m} = i, L_m = k\}$$

$$\pi_{2,i,k} = im_{n\to\infty} P\{P_m = 2, \xi_{2,m} = i, L_m = k\}$$

$$\pi_{3,i,k} = im_{n\to\infty} P\{P_m = 3, \xi_{3,m} = i, L_m = k\}$$

Multiply equation (6.2) both sides by z^k and taking summation over k and using (6.1) and on some algebraic simplifications,

$$\phi_{oi}(z) = \bar{\lambda}\phi_{oi+1}(z) + v_i\phi_{01}(z)\left[\frac{\bar{\lambda} + \lambda z}{z}\right] - \frac{\bar{\lambda}v_i}{z}\pi_{00} \tag{6.7}$$

Multiply equation (6.7) both sides by x^i and taking summation over i andafter some algebraic simplifications,

$$\phi_o(x,z)\left(\frac{x-\bar{\lambda}}{x}\right) = \phi_{01}(z)\left[\frac{(\bar{\lambda} + \lambda z)}{z}V(x) - \bar{\lambda}\right] - \frac{\bar{p}V(x)}{z}\pi \tag{6.8}$$

Multiply equation (6.4) both sides by z^k and taking summation over k and using (6.3),

$$\phi_{1i}(z) = \phi_{01}(z)a_i\bar{\lambda}(\bar{\theta} + \theta z) + \bar{\lambda}\phi_{i+1}(z) + \phi_{21}(z)a_i\bar{\lambda}(\theta z + \bar{\theta}) + \bar{\lambda}a_i\,\phi_{31}(z) - \lambda a_i\pi_{00} \tag{6.9}$$

Multiply equation (6.9) both sides by x^i and taking summation over i and after some algebraic simplifications,

$$\varphi_1(x,z)\left(\frac{x-\bar{\lambda}}{x}\right) = \varphi_{01}(z)\bar{\lambda}(\bar{\theta} + \theta z)(A(x) - a_0) - \bar{\lambda}\phi_{11}(z) + \phi_{21}(z)\bar{\lambda}(\bar{\theta} + \theta z)(A(x) - a_0)$$

$$+\phi_{31}(z)(A(x) - a_0) - \lambda\pi_{00}(A(x) - a_0) \tag{6.10}$$

Multiply Equation (6.5) both sides by z^k and taking summation over k and using (6.3),

$$\phi_{2i}(z)= ps_i\phi_{01}(z)(\bar{\lambda}+\lambda z)+\frac{\bar{\lambda}ps_i}{z}\phi_{11}(z)+(\bar{\lambda}+\lambda z)\phi_{2i+1}(z)+p\lambda s_i\phi_1(1,z)+ps_i\frac{(\bar{\theta}+\theta z)(\bar{\lambda}+\lambda z)}{z}\phi_{21}(z)$$

$$+\lambda ps_i\left(\frac{z-a_0}{z}\right)\pi_{00} \qquad (6.11)$$

Multiply equation (6.11) both sides by x^i and taking summation over i and after some algebraic simplifications,

$$\phi_2(x,z)\left(\frac{x-(\bar{\lambda}+\lambda z)}{x}\right)=(\bar{\lambda}+\lambda z)pS(x)\phi_{01}(z)+\frac{\bar{\lambda}pS(x)}{z}\phi_{11}(z)+pS(x)\left(\frac{a_0\bar{\lambda}+\lambda z}{z}\right)\varphi_{11}(z)$$

$$+\lambda pS(x)\varphi_1(1,z)+\left[\frac{(\bar{\theta}+\theta z)(a_0\bar{\lambda}+\lambda z)}{z}pS(x)-(\bar{\lambda}+\lambda z)\right]\varphi_{21}(z)$$

$$+\frac{\bar{\lambda}(z-a_0)}{z}pS(x)\pi_{00} \qquad (6.12)$$

In equation (6.6) multiply both sides by z^k and taking summation over k andusing (6.3)

$$\phi_{3i}(z)=\bar{p}r_i(\bar{\lambda}+\lambda z)\phi_{01}(z)+(\bar{\lambda}+\lambda z)\phi_{3i+1}(z)+\lambda z\bar{p}r_i\phi_1(1,z)+\bar{p}r_i(\bar{\lambda}a_0+\lambda z)\phi_{31}(z)$$

$$+(\bar{\lambda}a_0+\lambda z)(\bar{\theta}+\theta z)\bar{p}r_i\phi_{21}(z)\bar{p}\bar{\lambda}r_i\phi_{11}(z)+(\bar{\lambda}a_0+\lambda z)(\bar{\theta}+\theta z)\bar{p}r_i\phi \qquad (6.13)$$

Multiply equation (6.13) both sides by x^i and taking summation over i and after some algebraic simplifications,

$$\varphi_3(x,z)\left[\frac{x-(\bar{\lambda}+\lambda z)}{x}\right]=(\bar{\lambda}+\lambda z)\bar{p}R(x)\varphi_{01}(z)+\lambda z\bar{p}\phi\ (1,z)R(x)+\left[\bar{p}(\bar{\lambda}a_0+\lambda z)R(x)-(\bar{\lambda}+\lambda z)\right]\phi_{11}(z)$$

$$(6.14)$$

put x = 1 in (6.10) and on some algebraic simplifications,

$$\phi_1(1,z)\lambda=\bar{\lambda}(\bar{\theta}+\theta z)(1-a_0)\phi_{01}(z)-\bar{\lambda}\phi_{11}(z)+\phi_{21}(z)\bar{\lambda}(\bar{\theta}+\theta z)(1-a_0)+\bar{\lambda}(1-a_0)\phi_{31}(z)-\lambda(1-a_0)\pi_{00} \qquad (6.15)$$

using (6.15) in (6.12) and on some algebraic simplifications,

$$\phi_2(x,z)\left[\frac{x-(\bar{\lambda}+\lambda z)}{x}\right]=pS(x)\left[(\bar{\lambda}+\lambda z)+\bar{\lambda}(\bar{\theta}+\theta z)(1-a_0)\right]\varphi_{01}(z)+\bar{\lambda}S(x)\left(\frac{1-z}{z}\right)\phi_{11}(z)$$

$$+\left[\frac{z+\bar{\lambda}a_0(1-z)}{z}pS(x)\right]\phi_{31}(z)+\frac{\bar{\lambda}a_0(1-z)}{z}pS(x)\pi_{00}$$

$$+\left[\frac{\left(z+\bar{\lambda}a_0(1-z)\right)\left(\bar{\theta}+\theta z\right)}{z}pS(x)-\left(\bar{\lambda}+\lambda z\right)\right]\phi_{21}(z)$$

(6.16)

using (6.15) in (6.14) and on some algebraic simplifications,

$$\varphi_3(x,z)\left[\frac{x-\left(\bar{\lambda}+\lambda z\right)}{x}\right]=\left[\left(\bar{\lambda}+\lambda z\right)+z\bar{\lambda}\left(\underline{\bar{\theta}+\theta z}\right)(1-a_0)\right]\bar{p}R(x)\phi_{01}(z)+\left[\left[z+\bar{\lambda}a_0(1-z)\right]\bar{p}R(x)-\left(\bar{\lambda}+\lambda z\right)\right]$$

$$+\left[z+\bar{\lambda}a_0(1-z)\right]\left(\bar{\theta}+\theta z\right)\bar{p}R(x)\phi_{21}(z)+\overline{p\bar{\lambda}}R(x)(1-z)\phi_{11}(z)$$

(6.17)

put $x=\bar{\lambda}$ in (6.8) and solving for $\varphi_{01}(z)$,

$$\phi_{01}(z)=\frac{\bar{\lambda}V\left(\bar{\lambda}\right)}{\left[\left(\bar{\lambda}+\lambda z\right)V\left(\bar{\lambda}\right)-\bar{\lambda}z\right]}\pi_{00}$$

(6.18)

put $x=\bar{\lambda}$ in (6.8) and on simplification,

$$\bar{\lambda}\phi_{11}(z)=\bar{\lambda}\left(\bar{\theta}+\theta z\right)\left(A\left(\bar{\lambda}\right)-a_0\right)\varphi_{01}(z)+\bar{\lambda}\left(\bar{\theta}+\theta z\right)\left(A\left(\bar{\lambda}\right)-a_0\right)\varphi_{21}(z)+\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)\varphi_{31}(z)\pi_{00}$$

$$-\lambda\left(A\left(\bar{\lambda}\right)-a_0\right)\text{(6.19)}$$

in equation (6.16) put $x=\bar{\lambda}+\lambda z$ and solving for $\Phi_{2,1}(z)$,

$$\phi_{21}(z)=\frac{\lambda pS\left(\bar{\lambda}+\lambda z\right)\left[\left(\bar{\lambda}+\lambda z\right)+\bar{\lambda}\left(\bar{\theta}+\theta z\right)(1-a_0)\right]\phi_{01}(z)+pS\left(\bar{\lambda}+\lambda z\right)(1-z)\bar{p}\phi_{11}(z)+\left[z+\bar{p}a_0z-1\right]pS\left(\bar{\lambda}+\lambda z\right)\phi_{31}(z)+pS\left(\bar{\lambda}+\lambda z\right)\lambda a_0(z-1)\pi_{00}}{\left\{S\left(\bar{\lambda}+\lambda z\right)-\left[\left(z+\bar{p}a_0(1-z)\right)pS\left(\bar{\lambda}+\lambda z\right)\left(\bar{\theta}+\theta z\right)\right]\right\}}$$

(6.20)

in equation (6.17) put x $=\bar{\lambda}+\lambda z$ and solving for $\Phi_{3,1}(z)$,

$$\phi_{31}(z)=\frac{\left[\left(\bar{\lambda}+\lambda z\right)+z\bar{\lambda}\left(\bar{\theta}+\theta z\right)(1-a_0)\right]\bar{p}R\left(\bar{\lambda}+\lambda z\right)\phi_{01}(z)+\bar{p}R\left(\bar{\lambda}+\lambda z\right)\left(\bar{\theta}+\theta z\right)\left[z+\bar{\lambda}a_0(1-z)\right]\phi_{21}(z)+\overline{p\bar{\lambda}}R\left(\bar{\lambda}+\lambda z\right)(1-z)\phi_{11}(z)+\bar{p}R\left(\bar{\lambda}+\lambda z\right)(z-1)a_0\pi_{00}}{\left\{\left(\bar{\lambda}+\lambda z\right)-\left[\left(z+\bar{\lambda}a_0(1-z)\right)\bar{p}R\left(\bar{\lambda}+\lambda z\right)\right]\right\}}$$

(6.21)

using (6.21) in (6.20) and after some algebraic calculations,

$$\phi_{21}(z)=\left[\frac{\phi_{01}(z)\bar{\lambda}R(\bar{\lambda}+\lambda z)[A(z)+B(z)]\pi_{00}}{\begin{array}{l} z(\bar{\lambda}+\lambda z)-pS(\bar{\lambda}+\lambda z)(\bar{\theta}+\theta z)[C(z)]-[D(z)] \\ \bar{\lambda}(A(\bar{\lambda})-a_0)(1-z)pS(\bar{\lambda}+\lambda z)(1+z+\bar{\lambda}a_0(1-z))\bar{p}R(\bar{\lambda}+\lambda z)[E(z)]\end{array}}\right] \tag{6.22}$$

where

$$A(z)=\left\{\begin{array}{l} pS(\bar{\lambda}+\lambda z)\left[(\bar{\lambda}+\lambda z)-\bar{p}R(\bar{\lambda}+\lambda z)\right]\left[(\bar{\lambda}+\lambda z)-(1-z)\bar{\lambda}(A(\bar{\lambda})-a_0)\right] \\ \left[\begin{array}{l}\bar{\lambda}(\bar{\theta}+\theta z)(1-z)(A(\bar{\lambda})-a_0)(1+z\bar{\lambda}a_0(1-z))+\left[\begin{array}{l}(\bar{\lambda}+\lambda z) \\ +\bar{\lambda}z(\bar{\theta}+\theta z)(1-a_0)\end{array}\right] \\ \left[2z+\bar{\lambda}a_0(1-z)\right]\end{array}\right] \\ +\left[(\bar{\lambda}+\lambda z)+z\bar{\lambda}a_0(1-z))(\bar{\theta}+\theta z)\right]+\left[\begin{array}{l}((\bar{\lambda}+\lambda z)-(z+\bar{\lambda}a_0(1-z)))\bar{\lambda}(1-z))(\bar{\theta}+\theta z) \\ (A(\bar{\lambda})-a_0)\end{array}\right] \\ \bar{\lambda}(A(\bar{\lambda})-a_0)(1-z)pS(\bar{\lambda}+\lambda z)(1+z+\bar{\lambda}a_0(1-z))\bar{p}R(\bar{\lambda}+\lambda z)\end{array}\right\}$$

$$B(z)=\left\{\begin{array}{l}(z-1)\left[\lambda+z+\bar{\lambda}a_0(1-z))\right]pS(\bar{\lambda}+\lambda z)\bar{p}R(\bar{\lambda}+\lambda z)\left[a_0+\bar{\lambda}(A(\bar{\lambda})-a_0)\right] \\ \left[(\bar{\lambda}+\lambda z)-\bar{p}R(\bar{\lambda}+\lambda z)\right]\left[z+\bar{\lambda}a_0(1-z)\right]-(1-z)\bar{\lambda}(A(\bar{\lambda})-a_0)) \\ +\bar{p}R(\bar{\lambda}+\lambda z)(z-1)\bar{\lambda}(A(\bar{\lambda})-a_0)+\bar{\lambda}(A(\bar{\lambda})-a_0)(z-1) \\ \left[(\bar{\lambda}+\lambda z)-(z+\bar{\lambda}a_0(1-z))\right]pS(\bar{\lambda}+\lambda z)\bar{p}R(\bar{\lambda}+\lambda z)(1+z+\bar{\lambda}a_0(1-z))\bar{p}R(\bar{\lambda}+\lambda z)\end{array}\right\}$$

$$C(z)=\left[\begin{array}{l}(z+\bar{\lambda}a_0(1-z))(1+\bar{p}R(\bar{\lambda}+\lambda z))(z+\bar{\lambda}a_0(1-z))+\bar{\lambda}(A(\bar{\lambda})-a_0) \\ (1-z)(1+z+\bar{\lambda}a_0(1-z))\bar{p}R(\bar{\lambda}+\lambda z)\end{array}\right]$$

$$D(z)=\left[\bar{p}R(\bar{\lambda}+\lambda z)(\bar{\theta}+\theta z)(1-z)\bar{\lambda}(A(\bar{\lambda})-a_0)\right]1$$

$$E(z)=\left[(\bar{\lambda}+\lambda z)-(z+\bar{\lambda}a(1-z)\ )\right]+\left[(\bar{\lambda}+\lambda z)-(z+\bar{\lambda}p(1-z)\ )\right]\bar{p}R(\bar{\lambda}+\lambda z)(z+\bar{p}a(1-z)\ )$$

using (6.19) and (6.22) in (6.21) and after some algebraic simplifications,

$$\phi_{31}(z)=\left[\frac{\phi_{01}(z)[F(z)]+\phi_{21}(z)[G(z)]+[H(z)]\pi_{00}}{z[I(z)]-[J(z)]}\right] \tag{6.23}$$

$$F(z)=\left\{\begin{array}{l}\bar{p}R\left(\bar{\lambda}+\lambda z\right)\left[\begin{array}{l}\left(\bar{\lambda}+\lambda z\right)+z\bar{\lambda}\left(\bar{\theta}+\theta z\right)\left(1-a_0\right)\left[\left(\bar{\lambda}+\lambda z\right)-\left(z+\bar{\lambda}a_0(1-z)\right)\right]\bar{p}R\left(\bar{\lambda}+\lambda z\right)(1-z)\\ \bar{\lambda}\left(\bar{\theta}+\theta z\right)\left(A\left(\bar{\lambda}\right)-a_0\right)\left[z\left(\bar{\lambda}+\lambda z\right)-\left(z+\bar{\lambda}a_0(1-z)\right)pS\left(\bar{\lambda}+\lambda z\right)-\left(\bar{\theta}+\theta z\right)\right]\end{array}\right]\\ +\bar{\lambda}\left(\bar{\theta}+\theta z\right)\left(A\left(\bar{\lambda}\right)-a_0\right)z(1-z)+\left[\begin{array}{l}\left[\left(\bar{\lambda}+\lambda z\right)-\left(z+\bar{\lambda}a_0(1-z)\right)\right]\left(z+\bar{\lambda}a_0(1-z)\right)pS\left(\bar{\lambda}+\lambda z\right)\\ \left[\left(\bar{\lambda}+\lambda z\right)+\bar{\lambda}\left(\bar{\theta}+\theta z\right)\left(1-a_0\right)\right]\end{array}\right]\end{array}\right\}$$

$$G(z)=\left\{\begin{array}{l}\bar{\lambda}pS\left(\bar{\lambda}+\lambda z\right)(1-z)\bar{p}R\left(\bar{\lambda}+\lambda z\right)\left(\bar{\theta}+\theta z\right)\left\{(1-z)\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)+\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)\right\}\\ \bar{p}R\left(\bar{\lambda}+\lambda z\right)\left(z+a_0(1-z)\right)\left(\bar{\theta}+\theta z\right)\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)\end{array}\right\}$$

$$H(z)=\left\{\begin{array}{l}\lambda S\left(\bar{\lambda}+\lambda z\right)\bar{p}R\left(\bar{\lambda}+\lambda z\right)\left(\bar{\theta}+\theta z\right)(1-z)\left\{(1-z)\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)+\left[\left(\bar{\lambda}+\lambda z\right)-\left(z+\bar{\lambda}a_0(1-z)\right)\right]\right\}\\ \bar{p}R\left(\bar{\lambda}+\lambda z\right)\left(z+\bar{\lambda}a_0(1-z)\right)\left[a_0-\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)\right]\end{array}\right\}$$

$$I(z)=\left[\begin{array}{l}\left[\left(\bar{\lambda}+\lambda z\right)-\left(z+\bar{\lambda}a_0(1-z)\right)\right]\left(\left(z+\bar{\lambda}a_0(1-z)\right)-(1-z)\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)\right)\\ z\left(\bar{\lambda}+\lambda z\right)-\left(z+\bar{\lambda}a_0(1-z)\right)pS\left(\bar{\lambda}+\lambda z\right)-\left(\bar{\theta}+\theta z\right)\end{array}\right]$$

$$J(z)=\left[\begin{array}{l}pS\left(\bar{\lambda}+\lambda z\right)\bar{p}R\left(\bar{\lambda}+\lambda z\right)(1-z)\bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)+\left[\left(\bar{\lambda}+\lambda z\right)+\left(z+\bar{\lambda}a_0(1-z)\right)\right]\bar{p}R\left(\bar{\lambda}+\lambda z\right)\left(z+\bar{\lambda}a_0(1-z)\right)\\ \bar{\lambda}\left(A\left(\bar{\lambda}\right)-a_0\right)z\left[\bar{\theta}+\theta z\right]\bar{\lambda}\end{array}\right]$$

from equations (6.8), (6.10), (6.12) and (6.14) the PGF of the system size of the model under consideration is obtained from

$$\varphi(z)-\varphi_{0,1}(1,z)+\varphi_{1,1}(1,z)+\varphi_{2,1}(1,z)+\varphi_{3,1}(1,z)$$

$$P(z)=\phi_{01}(z)P_1(z)+\phi_{11}(z)P_2(z)++\phi_{21}(z)P_3(z)+\phi_{31}(z)P_4(z)-P_5(z)\pi_{00} \tag{6.24}$$

where $P_1(z)=\left[\dfrac{\bar{\lambda}(1-z)^2+z\left[1+\bar{\lambda}\left(\bar{\theta}+\theta z\right)\left((1-a_0)(2-z)+z\right)\right]}{\lambda z(1-z)}\right]$, $P_2(z)=\left[\dfrac{\bar{\lambda}(1-zp)}{\lambda}\right]$

$$P_3(z)=\left[\frac{\bar{\lambda}(1-a_0)}{\lambda}+\frac{\left[z+\bar{\lambda}a_0(1-z)\right]\left(\bar{\theta}+\theta z\right)\left(\bar{p}+pz\right)}{\lambda z(1-z)}-\frac{\left(\bar{\lambda}+\lambda z\right)}{\lambda(1-z)}\right]$$

$$P_4(z)=\left[\frac{\bar{\lambda}(1-a_0)}{\lambda}+\frac{\left(z+\bar{\lambda}a_0(1-z)\right)\left(\bar{p}+pz\right)}{\lambda z(1-z)}-\frac{\left(\bar{\lambda}+\lambda z\right)}{\lambda(1-z)}\right]$$

$$P_5(z)=\left[\frac{a_0\left(p\lambda+\bar{p}z\right)-\bar{\lambda}}{\lambda z}-\left(1-a_0\right)\right]$$

and $\varphi_{01}(z)$, $\varphi_{11}(z)$, $\varphi_{21}(z)$, $\varphi_{31}(z)$ are respectively given by the equations (6.18), (6.19) , (6.22) and (6.23).

4.1 Particular case

When the vacation time is zero then the PGF (6.24) of the model under consideration is reduced into

$$\phi(z)=\frac{A(\bar{\lambda})v(\bar{\lambda}+\lambda z)(1-z)\left[1-S(\bar{\lambda}+\lambda z)\theta\right]}{\left[vS(\bar{\lambda}+\lambda z)(\bar{\theta}+\theta z)+\bar{v}zR(\bar{\lambda}+\lambda z)\right]\left[z+(1-z)\bar{p}A(\bar{\lambda})\right]-z(\bar{\lambda}+\lambda z)}\pi_{00}$$

which is the PGF of discrete time retrial queue with starting failures, Bernoulli feedback and general retrial times.by Atencia *et al.* [13].

5. STEADY STATE CONDITION

The steady state condition for the model under consideration is given below

$$\pi_{00}=\frac{2\lambda\left[A(\bar{\lambda})-a_0\right]\left[\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}\right]\left[\bar{\lambda}-V(\bar{\lambda})\right]\left[2\bar{\lambda}(1+p)+(p\bar{p})^2\left[A(\bar{\lambda})-a_0\right]+\bar{\lambda}(1-\bar{\lambda}a_0-\bar{\lambda})\right]}{\begin{array}{l}2\lambda\left[A(\bar{\lambda})-a_0\right]\left[\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}\right]\left[\bar{\lambda}-V(\bar{\lambda})\right]\left[2\bar{\lambda}(1+p)+(p\bar{p})^2\left[A(\bar{\lambda})-a_0\right]+\bar{\lambda}(1-\bar{\lambda}a_0-\bar{\lambda})\right]\\ \left\{\left[\lambda V(\bar{\lambda})(2-a_0)(1-\bar{\lambda}a_0)+a_0\lambda\bar{\lambda}(1-p)\bar{\lambda}\left[A(\bar{\lambda})-a_0\right]\right]+\left[\bar{\lambda}-V(\bar{\lambda})\right]\right\}\\ +2\left[\bar{\lambda}-V(\bar{\lambda})\right]\left\{\begin{array}{l}2(\bar{\lambda})^2\left[A(\bar{\lambda})-a_0\right](\bar{p})^2\left(2\bar{\lambda}+2+2\bar{\lambda}p\right)+2(\bar{\lambda})^2V(\bar{\lambda})(1-a_0)^2\left[A(\bar{\lambda})-a_0\right]\\ +(\bar{\lambda})^2(1-a_0)p\bar{p}(2-\bar{\lambda}a_0-a_0)\end{array}\right\}\end{array}}$$

which is obtained from PGF by substituting z =1 and to equating to one and it is clearly less than one.

6 PERFORMANCE MEASURE

6.6.1 Expected queue length

Performance measure of the queue length distribution that is predictable queue length is attained below. The expected of the queue length is attained by differentiating the PGF (6.24) with respect to z and then put z=1,

$$E(Q)=\frac{\left[\bar{\lambda}-V(\bar{\lambda})\right]^{2}\left\{\begin{array}{l}2(\bar{\lambda})^{2}[A(\bar{\lambda})-a_{0}](\bar{p})^{2}(2\bar{\lambda}+2+2\bar{p})+(\bar{\lambda})^{2}v(\bar{\lambda})(1-a_{0})^{2}(A(\bar{\lambda})-a_{0})\\ +(\bar{\lambda})^{2}(1-a_{0})p\bar{p}(2-\bar{\lambda}a_{0}-a_{0})+2[A(\bar{\lambda})-a_{0}[2\bar{\lambda}(1+p)+(p\bar{p})^{2}[A(\bar{\lambda})-a_{0}]]\\ +[\bar{\lambda}[1-\bar{\lambda}a_{0}-\bar{\lambda}][\bar{p}V(\bar{\lambda})-\bar{p}][p\lambda\bar{\lambda}[A(\bar{\lambda})-a_{0}]+a_{0}\lambda(\lambda p+\bar{p})-(1-a_{0})]]\end{array}\right\}\pi_{00}+\left[\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}\right]\left\{\begin{array}{l}\bar{\lambda}V(\bar{\lambda})[A(\bar{\lambda})-a_{0}]^{2}[[V(\bar{\lambda})-\bar{\lambda}]\lambda(\bar{p}\theta-p)-\bar{\lambda}\bar{p}[[\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}]+[V(\bar{\lambda})-\bar{\lambda}]]]\\ 2\lambda[\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}[2\lambda(1+p)+(p\bar{p})^{2}[[A(\bar{\lambda})-a_{0}]+\bar{\lambda}(1-\bar{\lambda}a_{0}-\lambda)]]\\ +2\bar{\lambda}V(\bar{\lambda})[1+\bar{\lambda}(1+\theta)][\bar{\lambda}-V(\bar{\lambda})][A(\bar{\lambda})-a_{0}][\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}\left[2\bar{\lambda}(1+p)+(p\bar{p})^{2}\left[\begin{array}{l}[A(\bar{\lambda})-a_{0}]\\ +\bar{\lambda}(1-\bar{\lambda}a_{0})\end{array}\right]\right]\\ +[\bar{\lambda}-V(\bar{\lambda})]^{2}\left[\begin{array}{l}2(\bar{\lambda})^{2}[A(\bar{\lambda})-a_{0}](\bar{p})^{2}(2\bar{\lambda}+2+2\bar{p})+(\bar{\lambda})^{2}p(\bar{\lambda})(1-a_{0})^{2}(A(\bar{\lambda})-a_{0})\\ +(\bar{\lambda})^{2}(1-a_{0})p\bar{p}(2-\bar{\lambda}a_{0}-a_{0})\end{array}\right]\end{array}\right\}\pi_{00}}{[\bar{\lambda}-V(\bar{\lambda})][\bar{\lambda}V(\bar{\lambda})-\bar{\lambda}\left[2\lambda[A(\bar{\lambda})-a_{o}\left[2\bar{\lambda}(1+p)+(p\bar{p})^{2}[A(\bar{\lambda})-a_{0}]+\bar{\lambda}(1-\bar{\lambda}a_{0}-\bar{\lambda})\right]\right]}$$

7 NUMERICAL ILLUSTRATION

This section has numerical illustrations are briefly analyzed in two different cases. In both of thesetwo cases expected queue length is investigated in following manner.

1. The result on expected queue length when the arrival rate is increases
2. The result on expected queue length when the service rate is increases

Case 7.7.1

In this case customer arrival time, service time and vacation time are all geometrically distributed. When the customer arrival rate increases the result on expected queue length is investigated below with the following figure with table and assumed values given in the table.

Table 7.1 Arrival rate (λ) Vs.Expected queue length E(Q)

(λ)	**(p)**	(a_0)	$\boldsymbol{\pi_{00}}$	**E(Q)**
0.2	0.4	0.6	0.16	0.231
0.4	0.4	0.6	0.33	0.456
0.6	0.4	0.6	0.48	0.837
0.8	0.4	0.6	0.63	1.558
1.0	0.4	0.6	0.86	2.130

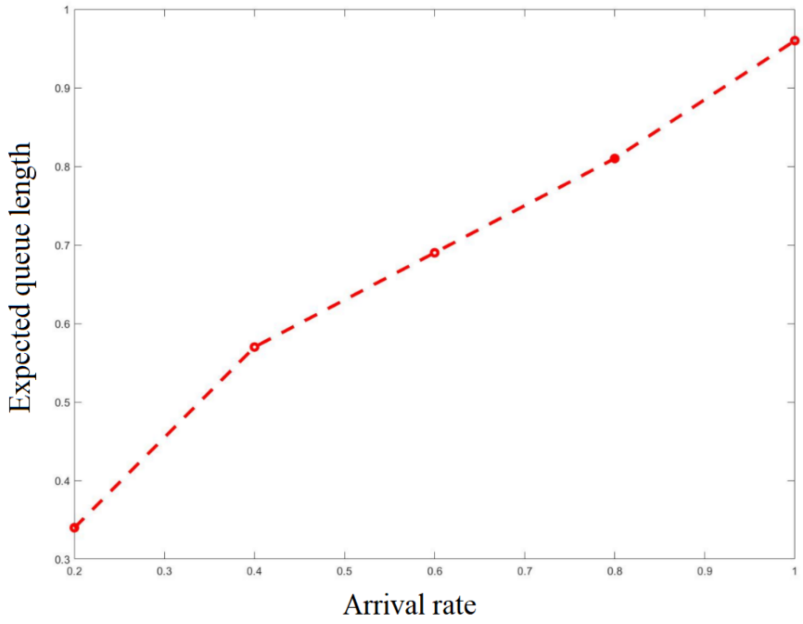

Figure 7.2 Arrival rate Vs. Expected queue length

It is noticed that expected queue length increases when arrival rate increaseswhich is inferred by above figure and table.

When the service rate increases the result on expected queue length is investigated below with the following figure with table.

Table7.2 Service rate (p) Vs.Expected queue length E(Q)

(p)	**(λ)**	(a_0)	π_{00}	**E(Q)**
0.2	0.4	0.6	0.74	0.178
0.4	0.4	0.6	0.61	0.142
0.6	0.4	0.6	0.42	0.044
0.8	0.4	0.6	0.32	0.031
1.0	0.4	0.6	0.21	0,004

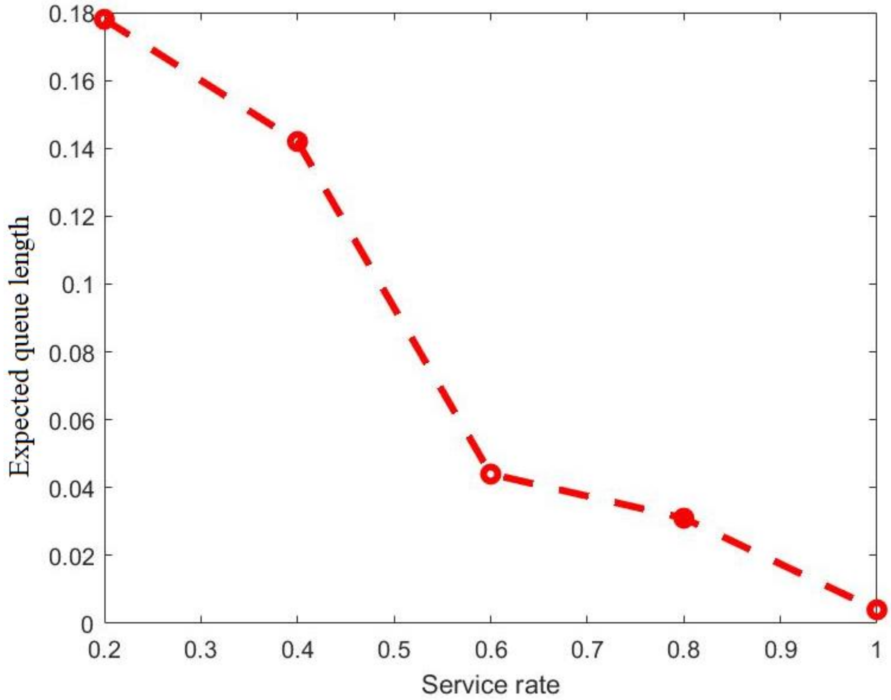

Figure 7.3 Service rate Vs. Expected queue length

It is noticed that expected queue length decreases when service rate increases which is inferred by above figure and table.

Case 7.7.2

Here customer arrival time obeys geometric distribution and. service time and vacation time are obeys poisson distribution.

When the customer arrival rate increases the result on expected queue length is investigated below with the following table with figure.

Table 7.3 Arrival rate (λ) Vs.Expected queue lengthE(Q)

(λ)	**(p)**	(a_0)	π_{00}	**E(Q)**
0.2	0.4	0.6	0.32	0.086
0.4	0.4	0.6	0.48	0.294
0.6	0.4	0.6	0.68	0.792
0.8	0.4	0.6	0.73	1.470
1.0	0.4	0.6	0.86	2.260

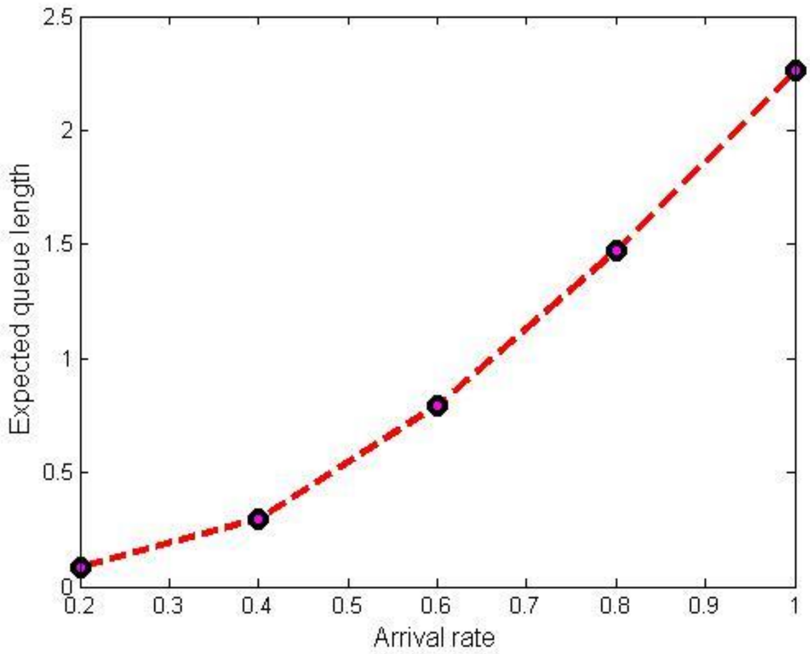

Figure 7.4 Arrival rate Vs. Expected queue length

It is noticed that expected queue length increases when arrival rate increases which is inferred by above figure and table.

When the service rateincreases the result on expected queue length is investigated below with the following table with figure.

Table 7.4 Service rate (p) Vs.Expected queue length E(Q)

(p)	**(λ)**	(a_0)	$\boldsymbol{\pi_{00}}$	**E(Q)**
0.2	0.4	0.6	0.92	0.298
0.4	0.4	0.6	0.75	0.293
0.6	0.4	0.6	0.61	0.290
0.8	0.4	0.6	0.45	0.267
1.0	0.4	0.6	0.27	0,158

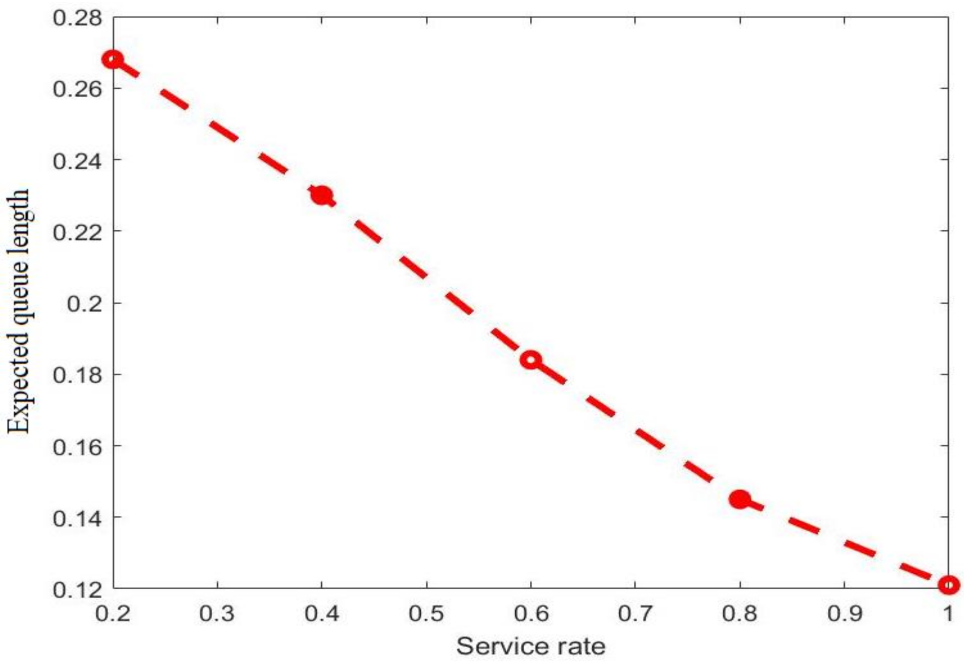

Figure 7.5 Service rate Vs. Expected queue length

It is noticed that expected queue length decreases when service rate increases which is inferred by above figure and table.

8.CONCLUSION

In this article a discrete time retrial queuing system with starting failures, Bernoulli feedback, general retrial times and a vacation has been analyzed in brief. In this model an analytical expression for PGF is derived by using generating function technique. In performance measure an expected queue length is derived in analytical expression form and by using this expression we investigate the length of the queue in several ways. In many real life situation this model is most applicable.

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